A Pop-up-Presentation on Maximum Likelihood Trend Estimation for Environmental Data

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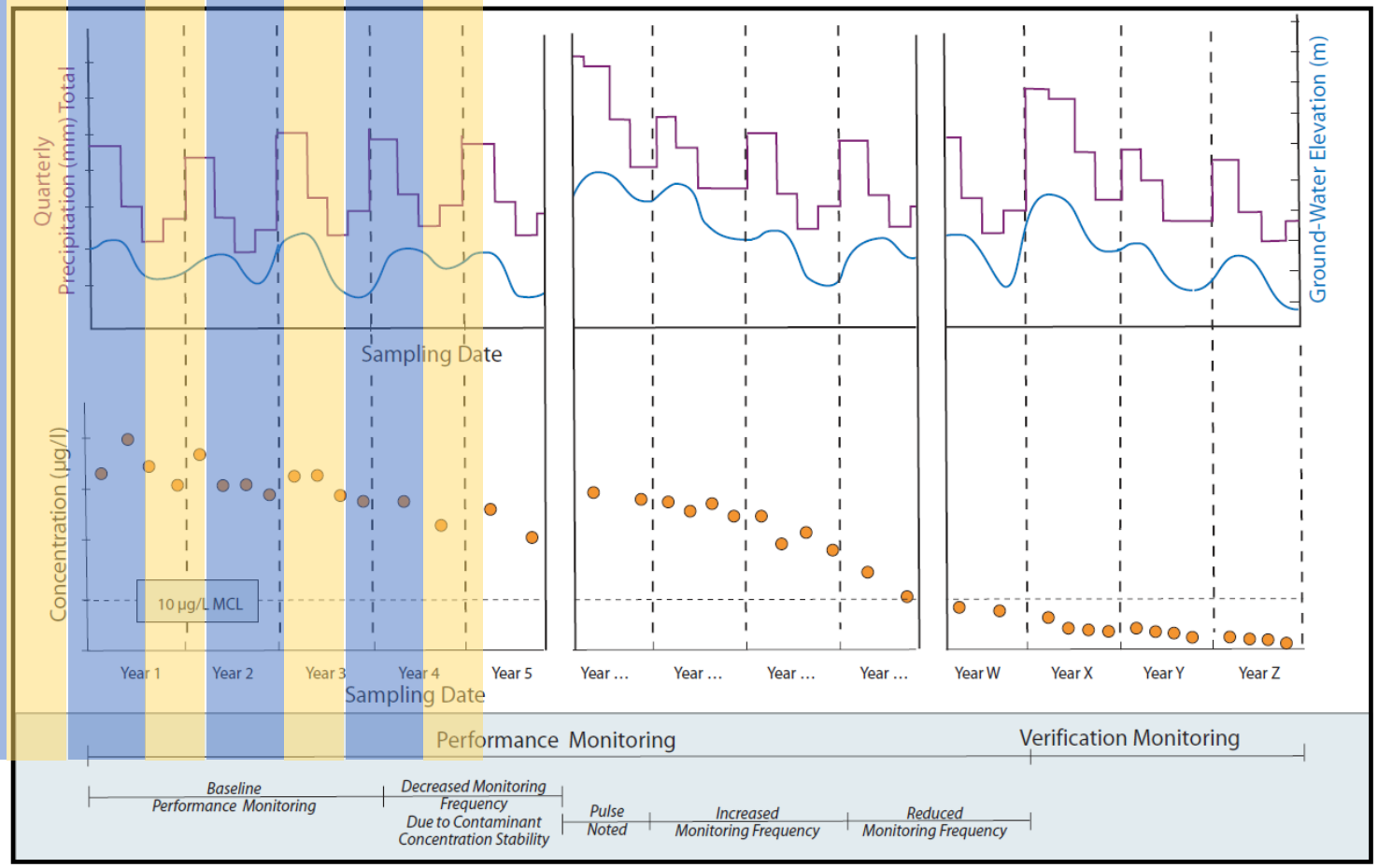
Some Shortcomings of Common Trend Methods

Parametric least-squares:

- Biased or just unusable with censored (non-detect) data

Non-parametric methods:

- Unable to incorporate additional explanatory variables (co-variates)
- Split into “seasonal subsets”



Pope, 2004

Univariate vs Multivariate Trend Analysis

Why does multi-variate matter?

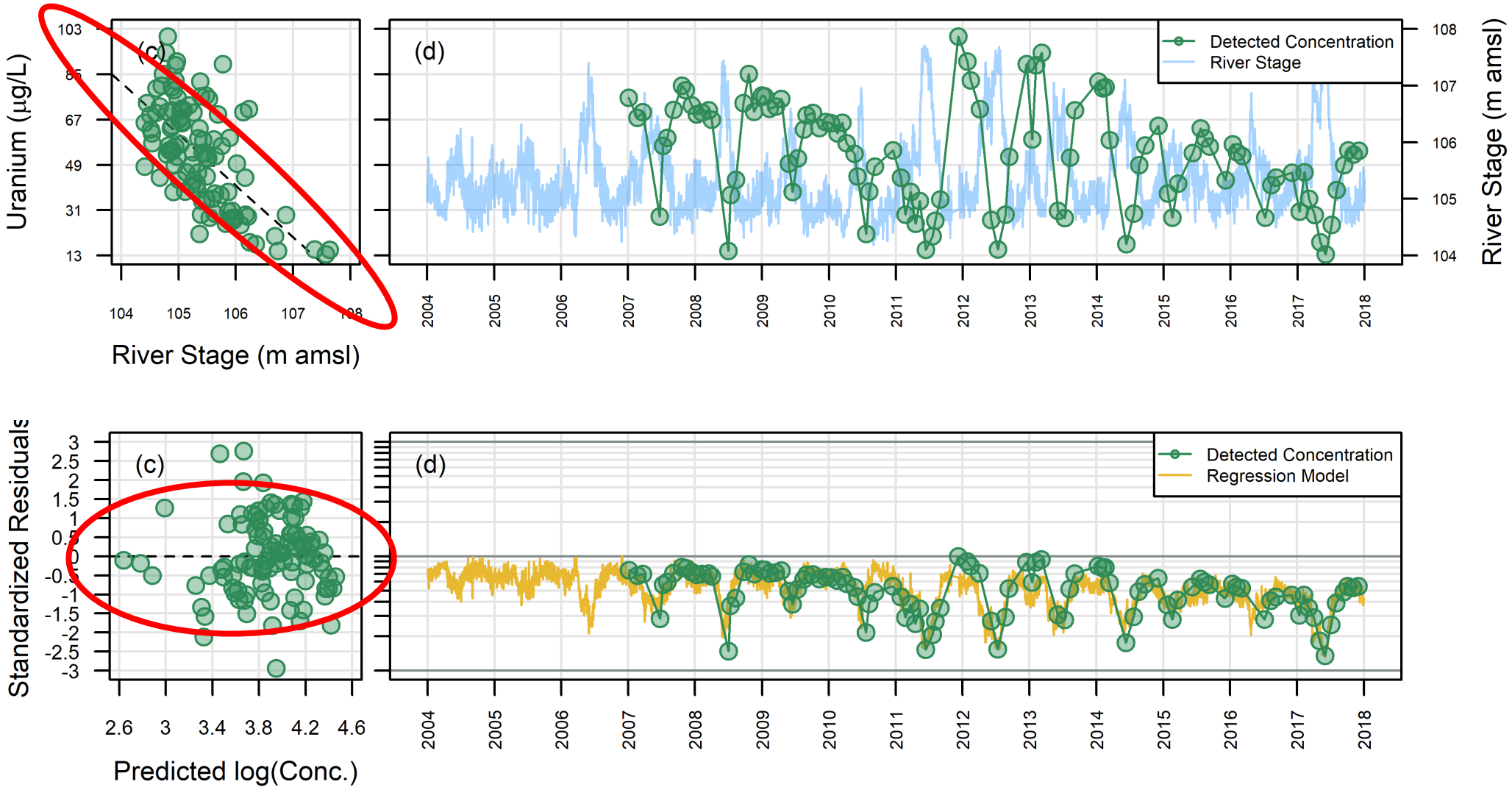
- Greater proportion of variability is properly ascribed to physical processes rather than to randomness
- Narrower confidence limits on estimates
- Greater power when testing and validating the underlying model

Maximum Likelihood [MLE] to the Rescue

Why does MLE matter?

- Both real and perceived advantages of OLS fall apart with censored data, largely due to arbitrary data substitution
- Stable multi-variate analysis is possible with MLE even when non-detects are present (*up to a point of course*)

An Example - Uranium



Another Example

Narrow confidence limit on trended mean despite high sample variance

In Summary

Non-Censored Datasets

- All data above reporting limits:
  - Univariate Methods:
    - OLS
    - Mann-Kendall/Thiel-Sen
  - Multivariate Methods:
    - OLS
    - MLE - Tobit Method

Censored Datasets

- Some data below reporting limits (i.e., non-detects)
  - Univariate Methods:
    - Mann-Kendall/Thiel-Sen
  - Multivariate Methods:
    - MLE - Tobit Method

DiFilippo, Tonkin, and Huber (2023). Use of Censored Multiple Regression to Interpret Temporal Environmental Data and Assess Remedy Progress (Accepted: Groundwater)