

# Delayed Subsidence in IWFM

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# Outline

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- Background and Objective
- Implementation in IWFM
- Examples
- Discussion
- Path forward

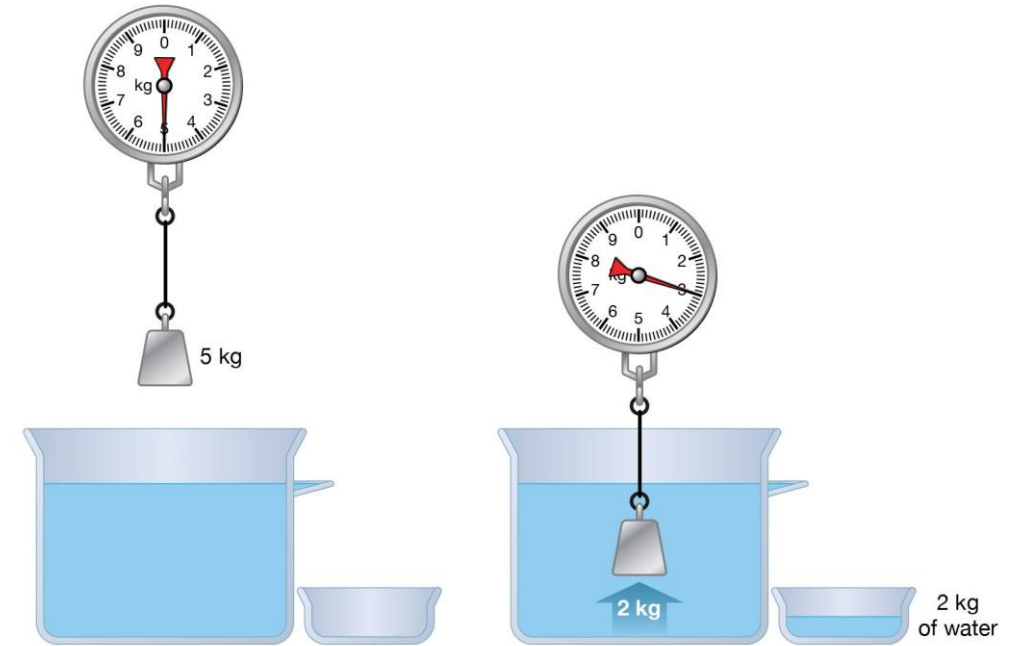
# Background and Objective

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# Underlying Principles

- Archimedes Principle
  - Volume of displaced fluid is equivalent to the volume of object fully immersed
  - Weight of the displaced portion of the fluid is equivalent to the magnitude of the buoyant force

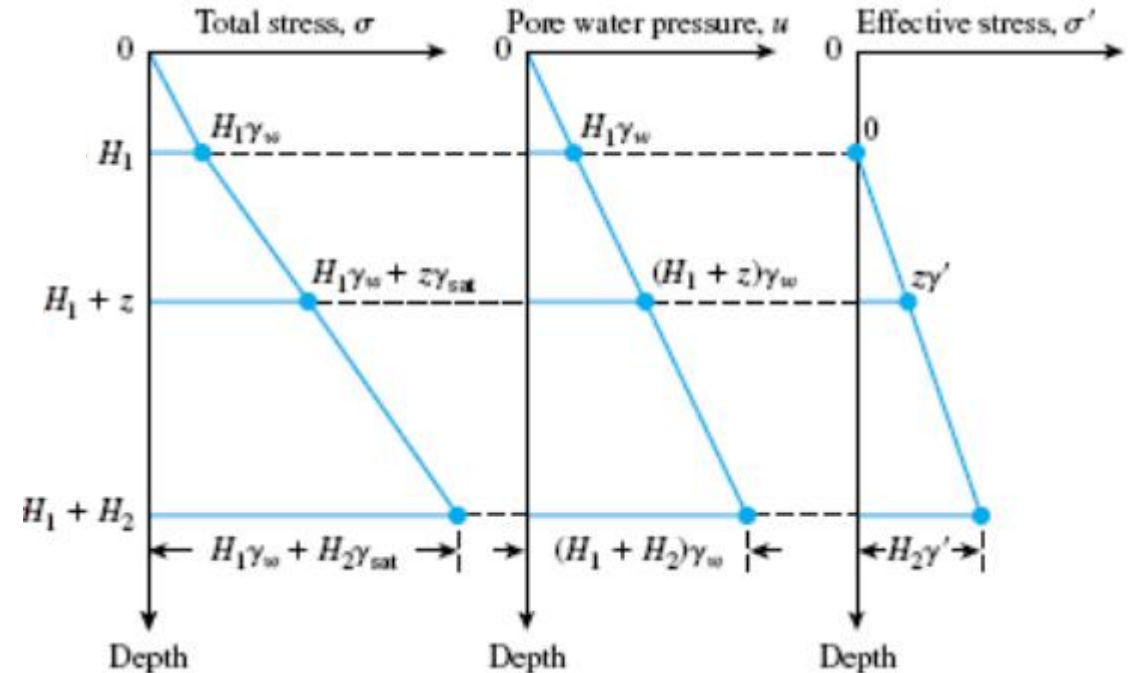
Archimedes' principle



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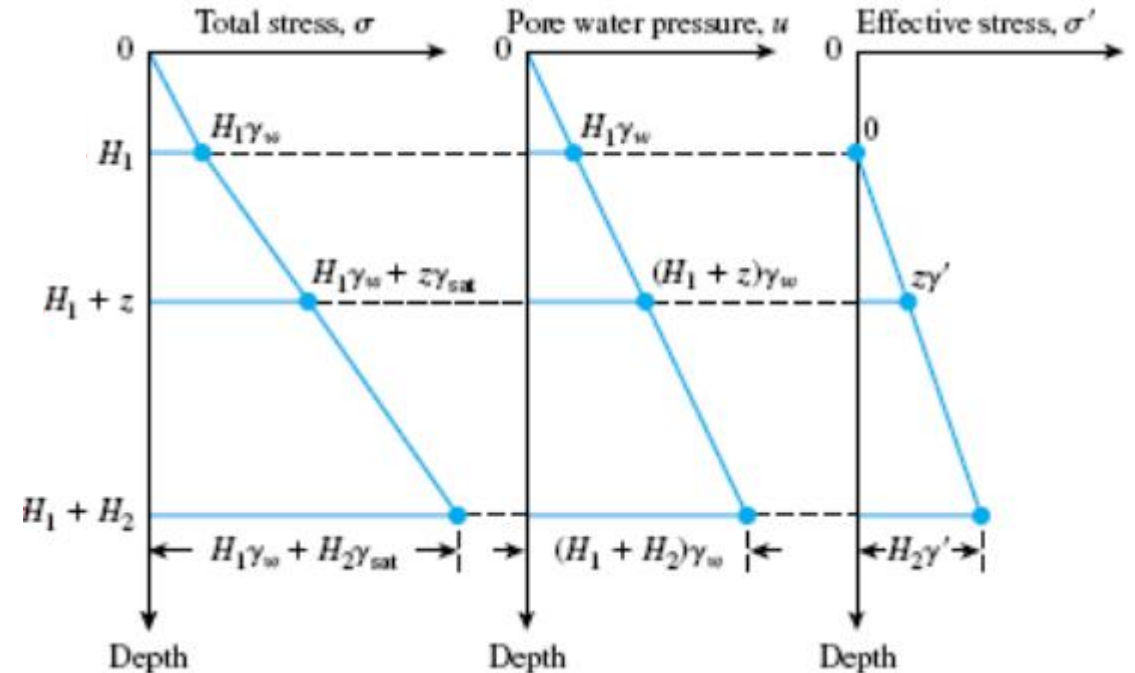
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- Terzaghi's Principle
  - When stress is applied to a porous material, it is opposed by the fluid pressure filling the pores of the material
  - Effective stress represents the average stress carried by the soil skeleton



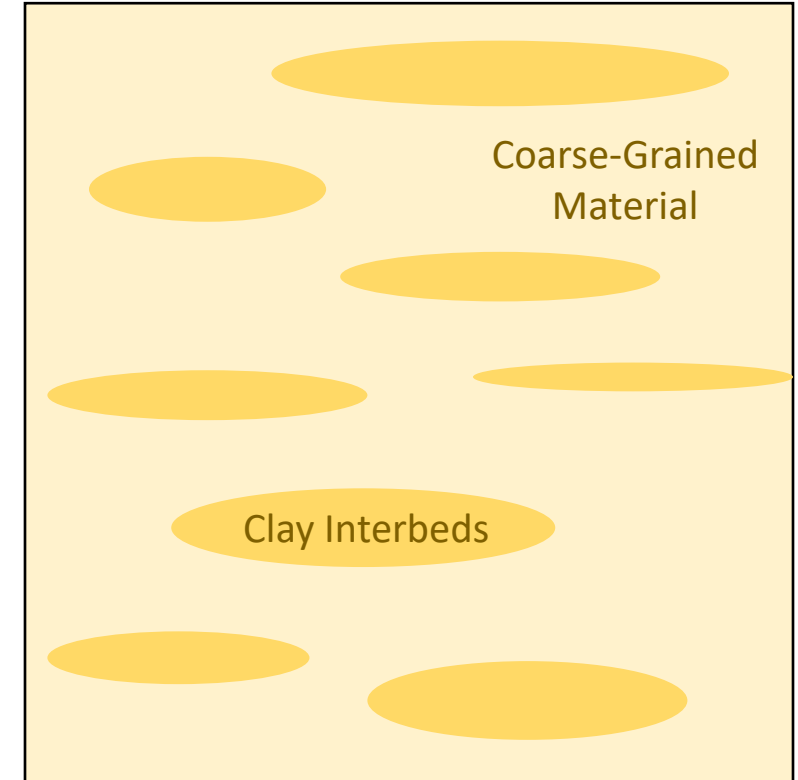
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  - Effective stress represents the average stress carried by the soil skeleton
- Pumped water reduces pore water pressure leading to consolidation resulting in land subsidence



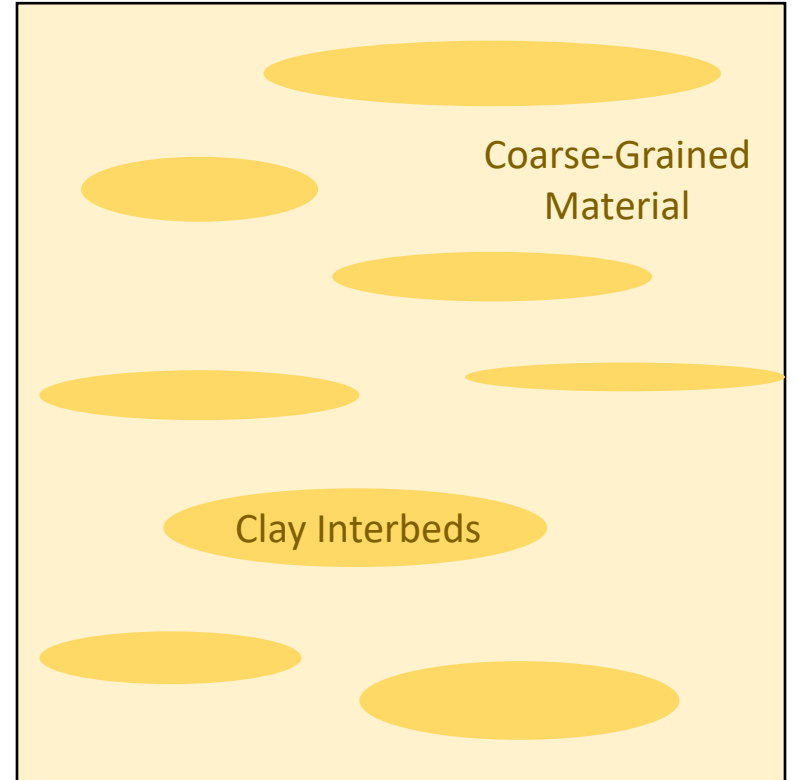
# Confined Storage

- Sand drains quickly and exhibits elastic storage comprised of water and soil compressibility
  - Reversible
  - Instantaneous



# Confined Storage

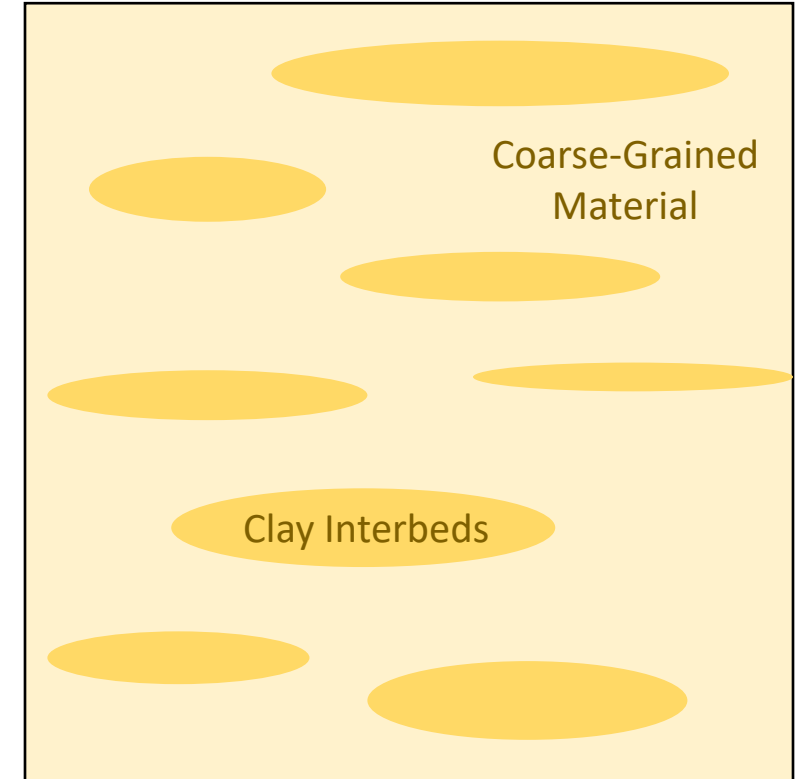
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- Clay interbeds:
  - Exhibit both elastic and inelastic storage





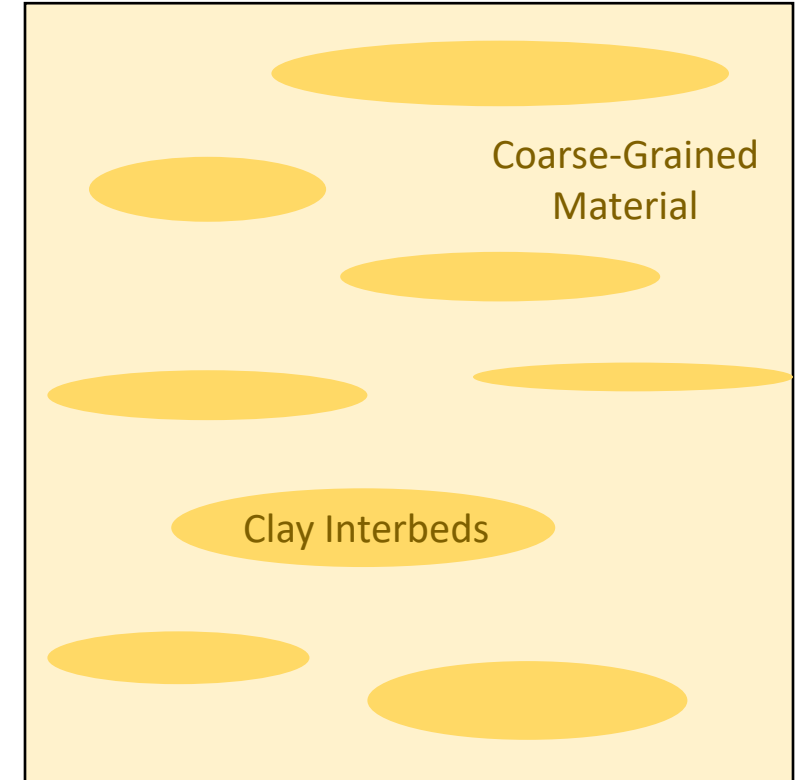
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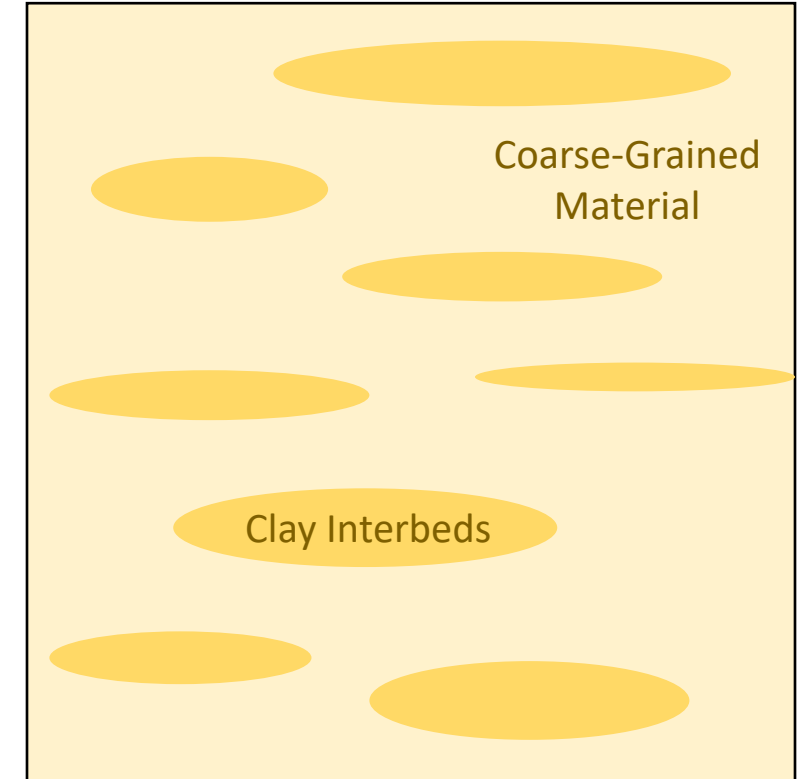
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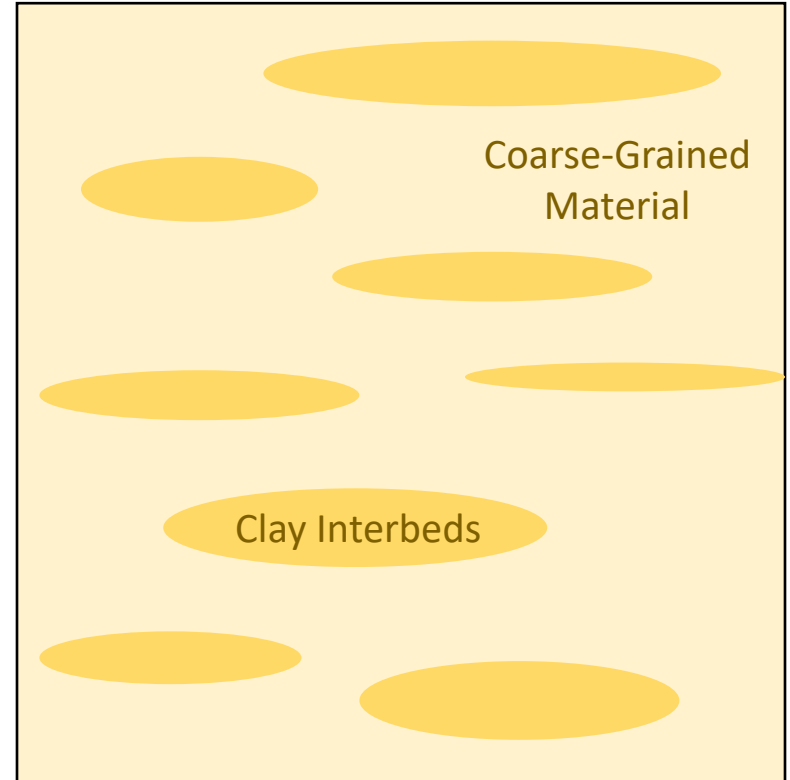
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  - Often ignore water compressibility and primarily deals with only soil skeletal compressibility
  - Exhibit a delayed decrease in pore water pressure
- Inelastic storage of fine-grained sediments can be one to two orders of magnitude larger than elastic storage



# IWFM

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- IWFM's subsidence formulation assumed instantaneous change in interbed storage

# Objective

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- Incorporate a time-delayed subsidence formulation in IWFM to accommodate the gradual increase in effective stress within interbeds

# Implementation in IWFM

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# Literature

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- Aquifer drainage model (1-D)
  - Terzaghi, 1925: forms the basis of all the theory developed later
  - Jacob, 1940: proposed a vertical one-dimensional model
  - Helm, 1976: provides theory assuming only one-dimensional vertical deformation used later in the SUB package of MODFLOW.
  - Supported by available parameter data
  - Most widely used
  - Implemented in all recent versions of MODFLOW



# Literature

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- Poroelasticity model (3-D)
  - Biot, 1941: provides the three-dimensional consolidation formulation (linear poroelasticity model). This has been used by code BIOT2D (Hsieh, 1996). This theory:
    - Considers horizontal deformation as well
    - Is much more complex
  - Rice and Cleary, 1976: derived a three-dimensional formulation that can be implemented in codes like MODFLOW and IWFM
  - Hsieh, 1996:
    - Developed an axisymmetric poroelasticity model, which was called Hsieh Displacement Model (HDM)
    - Compared to a three-dimensional model implemented in MODFLOW, which was called the Granular Displacement Model (GDM), based on Rice and Cleary (1976).
  - Provides better ability to locate potentially damaging fissures
  - The stress-strain constitutive relationships are cumbersome to implement in 3-D MODFLOW or IWFM models

# Literature

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- Poroviscosity model (secondary stress)
  - Incorporates secondary stress commonly used in geotechnical/foundation engineering, which other subsidence models ignore
  - Kooi et al, 2017: SUB-CR version of subsidence package uses the poroviscosity model and considers secondary compression or creep
- Example codes with rigorous stress-strain formulation for fracture analysis with thermo-hydro-mechanical (THM) coupling
  - COMSOL Multiphysics
  - TOUGH2
  - FEHM – hydrofracking, subsidence
  - HydroGeoSphere
  - SUTRA – Reeves et al 2000
  - FEFLOW

# MODFLOW Development

Package Name	MODFLOW Version	Notes	Reference
Interbed Storage (IBS1)	MODFLOW-2000	Assumes instantaneous storage change	Leake and Prudic (1991)
Interbed Storage (IBS2)	MODFLOW-2000	Not released publicly but used internally within the USGS	Leake (1990)
Subsidence and Aquifer-System Compaction (SUB)	MODFLOW-2000	Considers time-delayed storage change	Hoffmann et al (2003)
Subsidence and Aquifer-System Compaction Package for Water-Table Aquifers (SUB-WT)	MODFLOW-2005	Considers impact of water table changes	Leake and Galloway (2007)
Subsidence and aquifer system compaction that includes creep (SUB-CR)	MODFLOW-2005	Developed by Deltares; Considers secondary compression or creep	Kooi et al (2017)
Skeletal storage, compaction and subsidence (CSUB)	MODFLOW 6	Considers time-delayed storage change; creep not incorporated	Hughes et al (2022)

# Mathematical Formulation

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- Change in thickness can be expressed as:

$$\Delta b = \Delta h S_{sk} b_o$$

where,

$\Delta b$  is the change in thickness of the sediment layer [L],

$\Delta h$  is the change in hydraulic head [L],

$S_{sk}$  is the skeletal specific storage [1/L], and

$b_o$  is the initial thickness of the interbed.

$$S'_k{}^m = \begin{cases} S'_{ke} & \text{for } h_i^m > H_i^{m-1} \\ S'_{kv} & \text{for } h_i^m \leq H_i^{m-1} \end{cases}$$

# Mathematical Formulation

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- Governing equation within clay interbeds:

$$\frac{\partial^2 h}{\partial z^2} = \frac{S'_s}{K'_v} \frac{\partial h}{\partial t}$$

where,

$z$  is the vertical spatial coordinate [L],

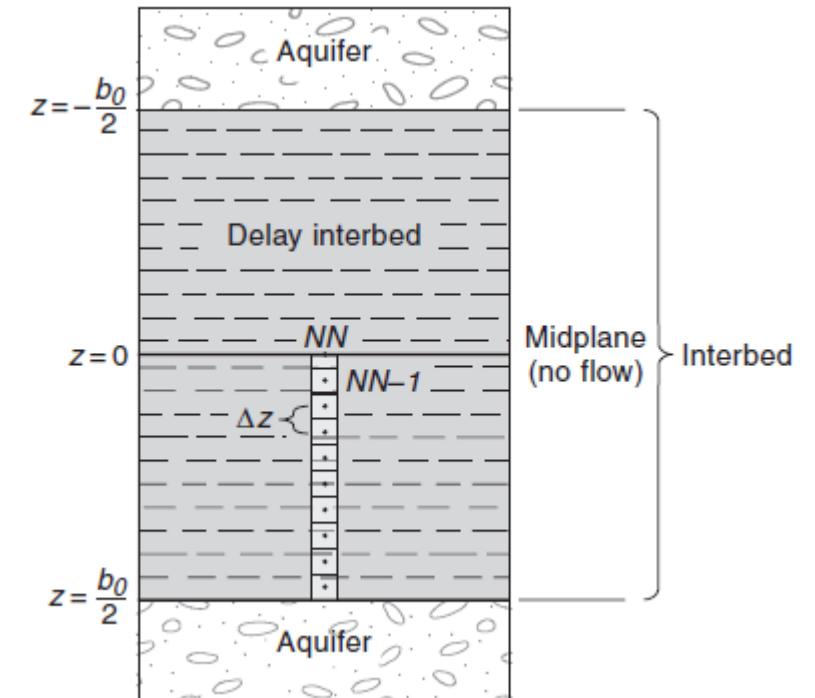
$S'_s$  is the specific storage of the interbed [1/L],

$K'_v$  is the vertical hydraulic conductivity of the interbed [L/T], and

$t$  is time [T].

# Numerical Implementation

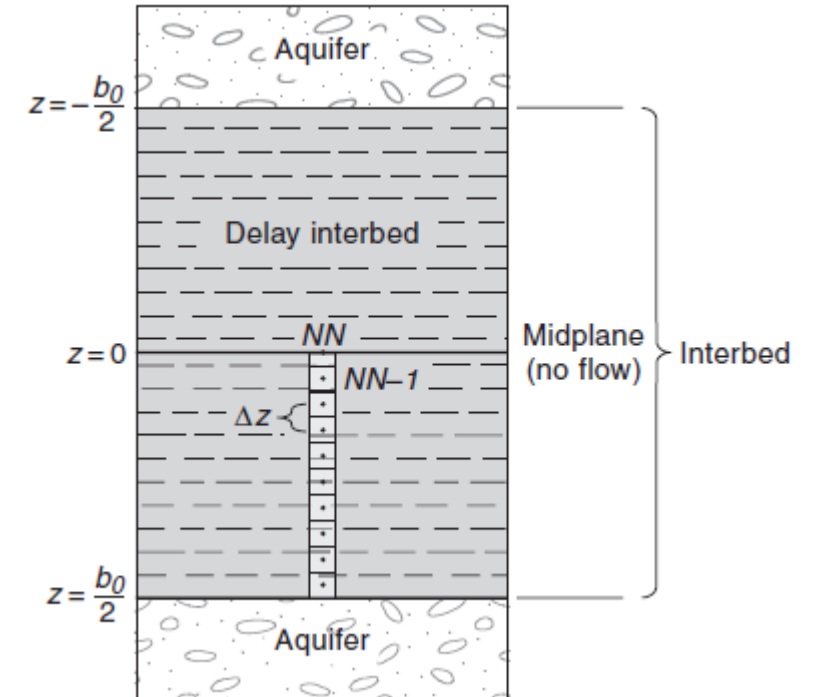
- Clay interbeds assumed symmetrical and only half thickness is solved
- Clay interbeds are discretized vertically
- Heads within clay interbeds solved using 1-D finite difference approximation
- Flow between the interbed and the surrounding aquifer is assumed to be vertical and caused by the hydraulic gradient across the horizontal interface of the interbed
- Flow in/out of interbed storage is treated as a flux term to the groundwater flow solution
- The two systems – interbeds and aquifer flow – simulated as a coupled solution



*Hoffmann et al (2003)*

# Numerical Implementation

- For each model cell, number of interbeds is an input to the model
- All interbeds within a layer are assumed to have a common equivalent thickness
- Number of sublayers (NN) is calculated separately for each model node
  - A preferred initial  $dz$  is an input to the model
  - Number of sub-layers (NN) is internally calculated
  - NN is held constant throughout the simulation
  - A minimum thickness is input to the model below which compaction is assumed negligible
- Land subsidence is calculated but model grid is not 'physically' changed



*Hoffmann et al (2003)*

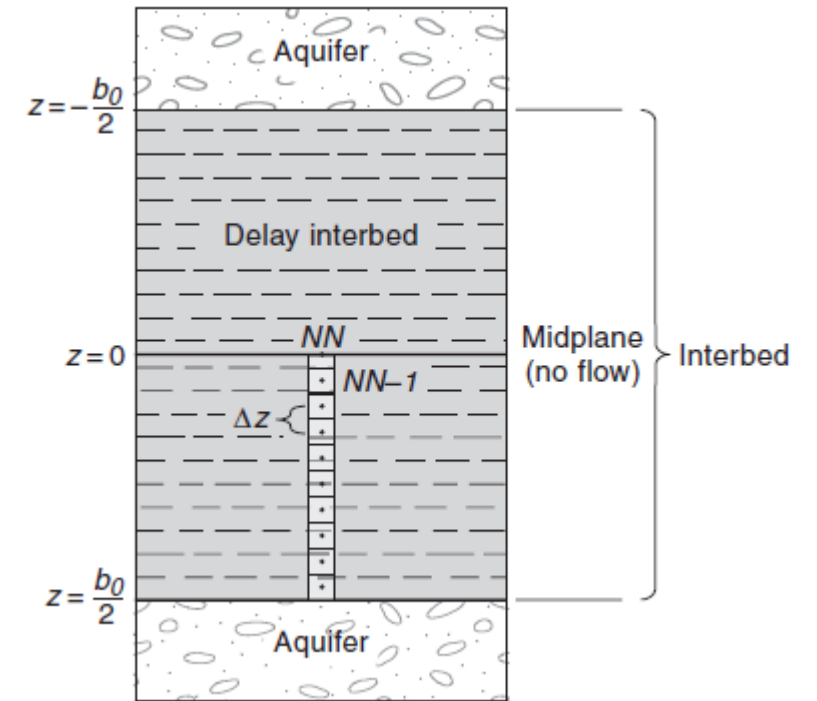
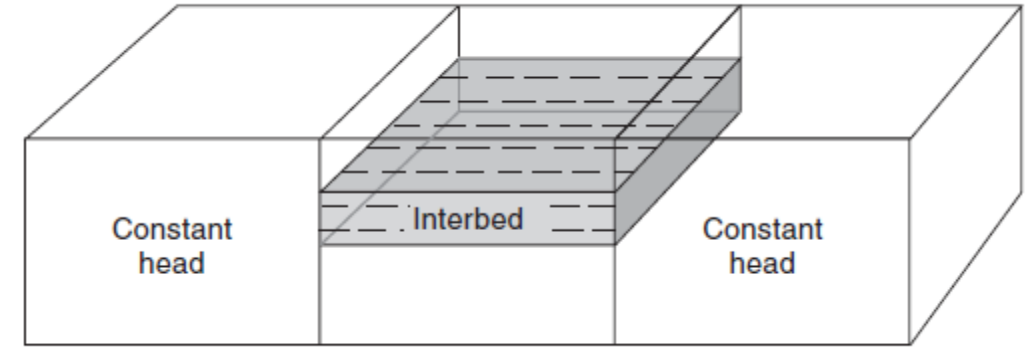
# Examples

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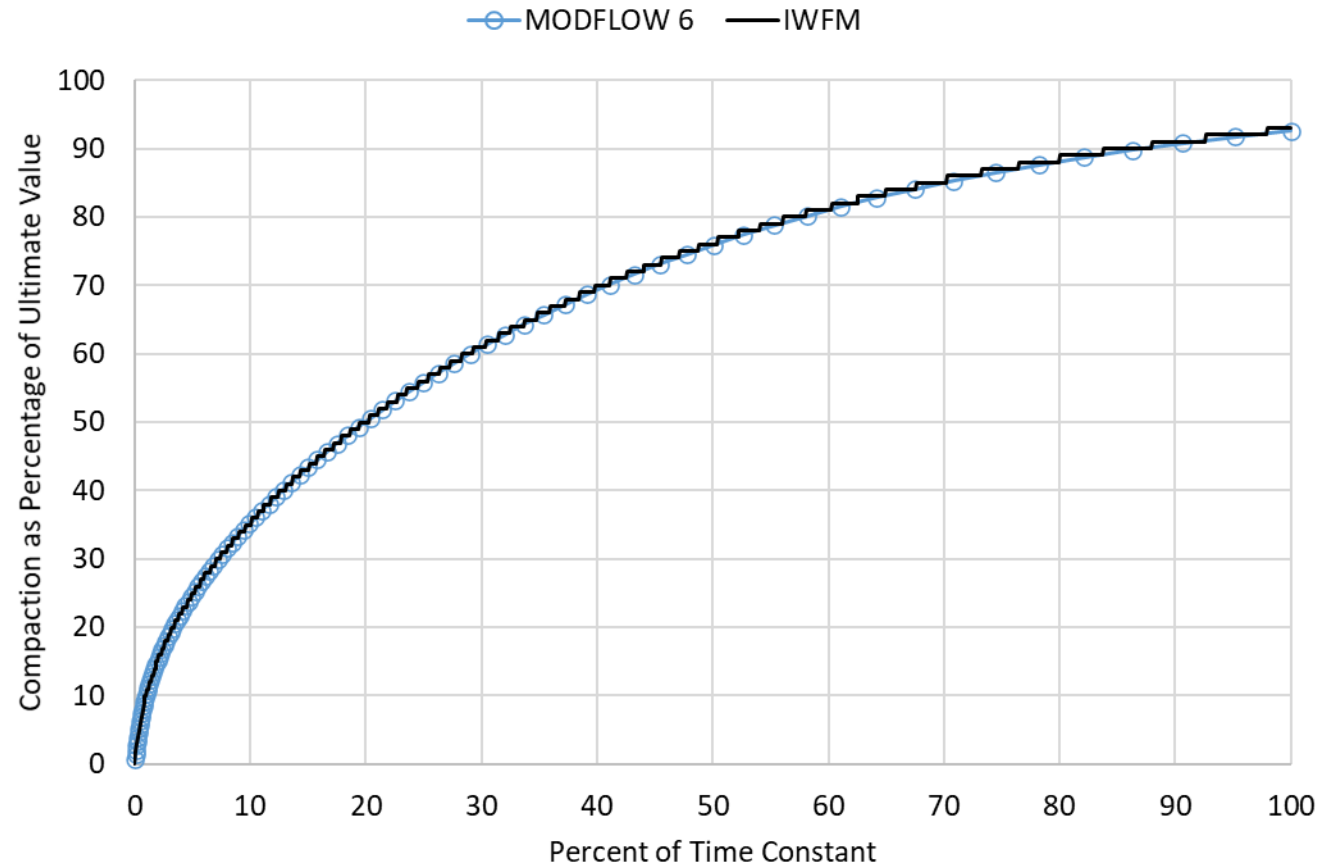
# Example 1

- Drainage of thick interbed
- Analytical solution is available
- Example presented in MODFLOW



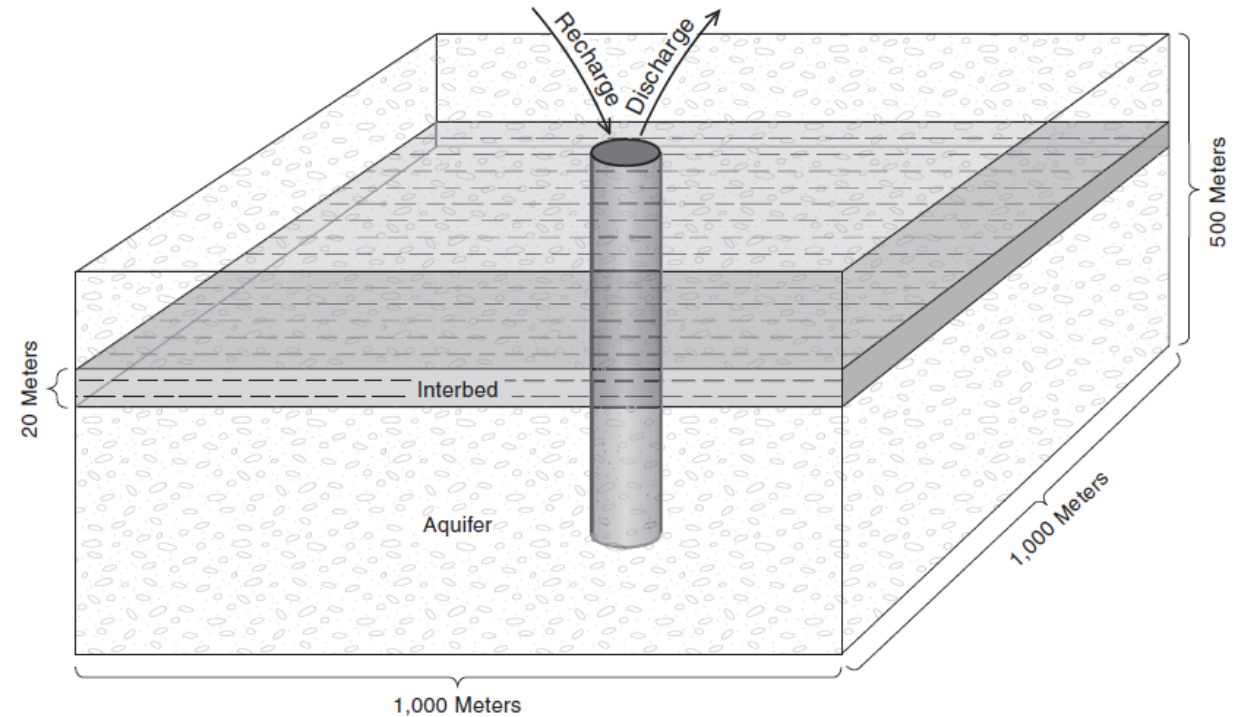
# Example 1

- IWFM matches the analytical solution and the MODFLOW results
- Analytical solution not shown here



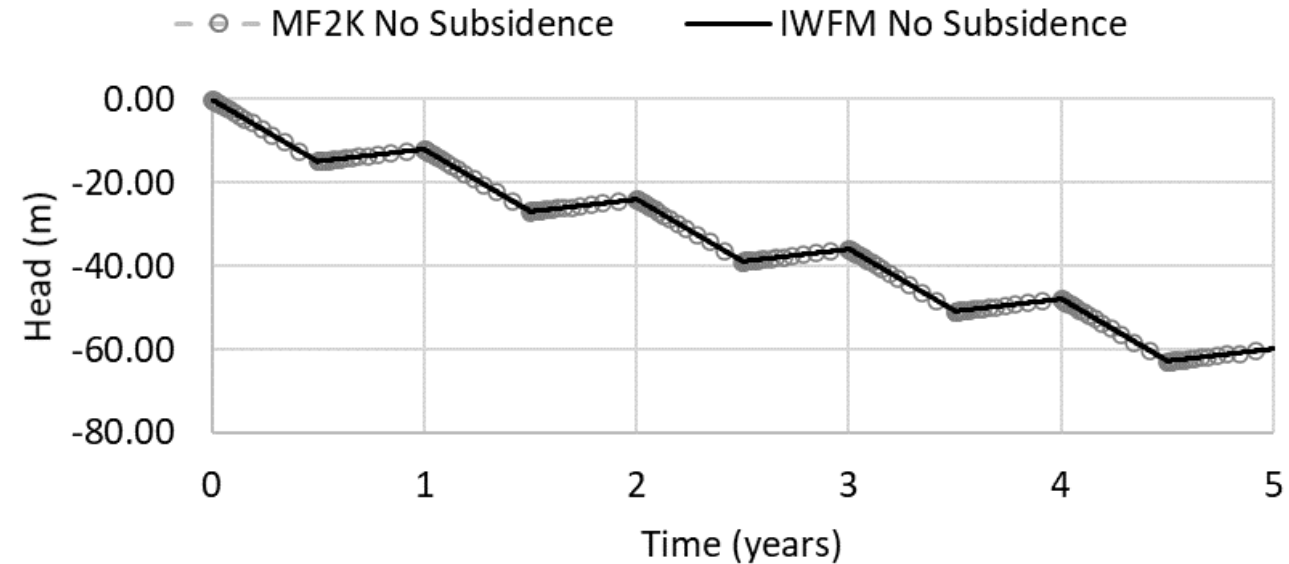
# Example 2

- Effects of seasonal pumping
- Results compared to MODFLOW
- Compared both no-delay and with-delay formulations



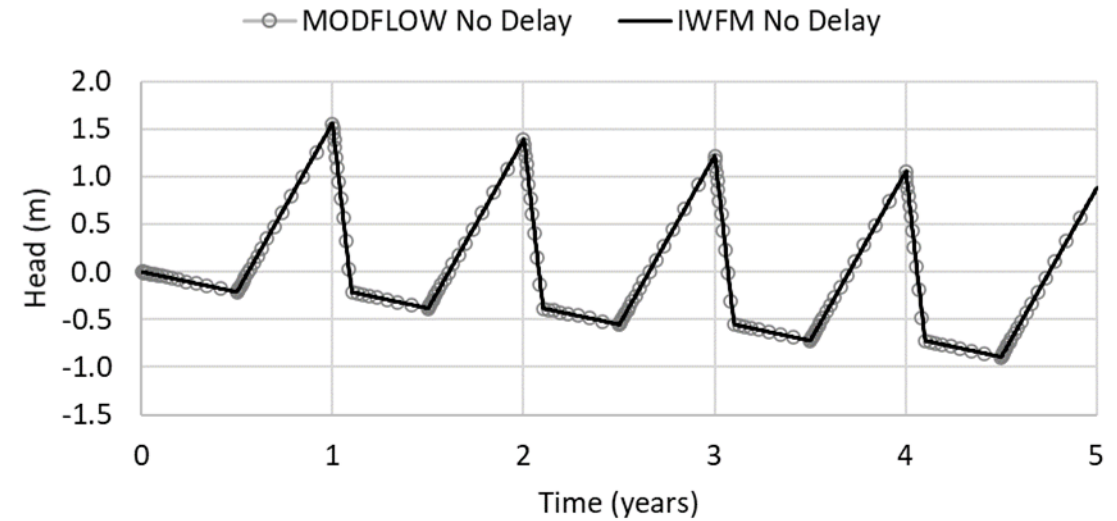
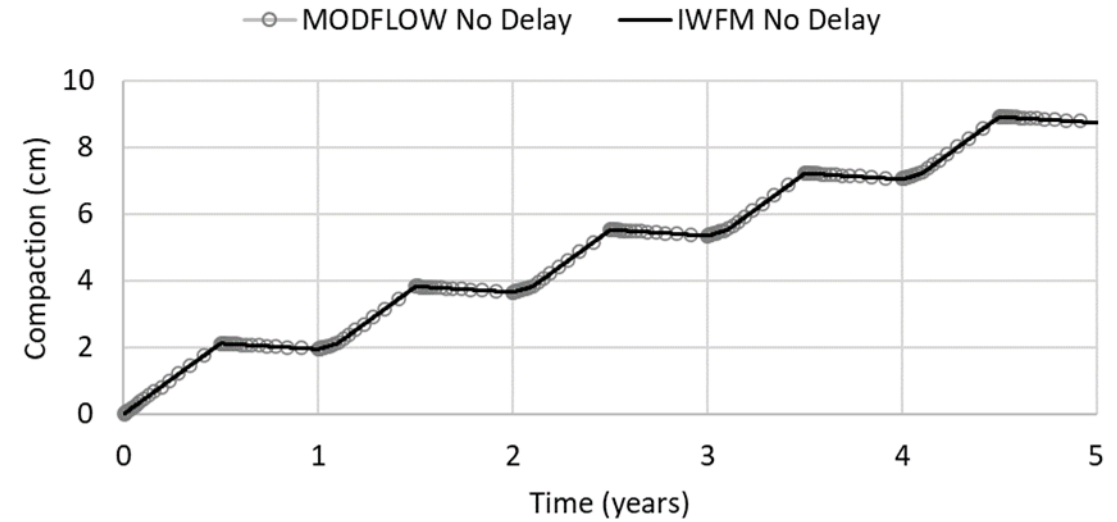
# Example 2

- MODFLOW and IWFM models compared without subsidence to ensure same model was simulated



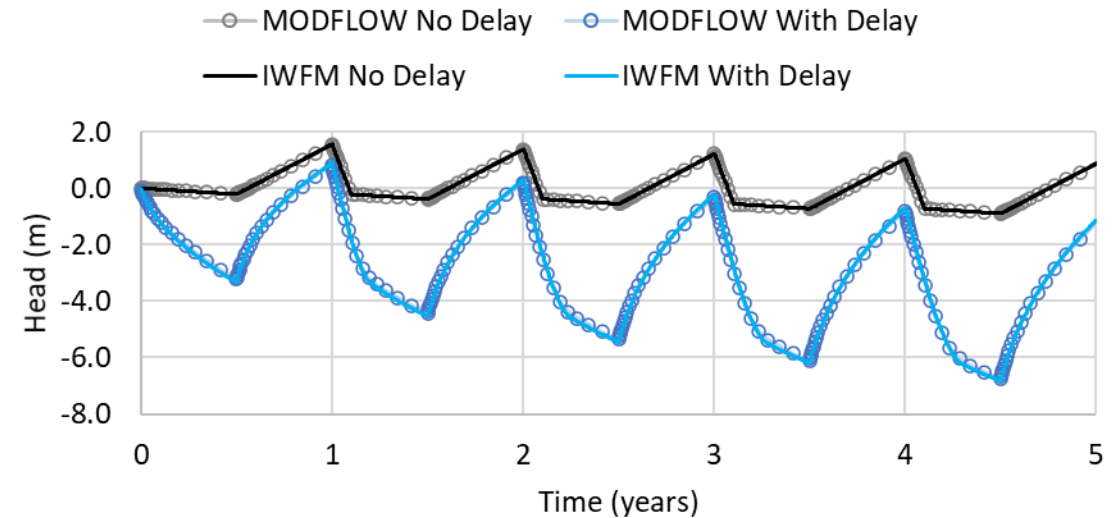
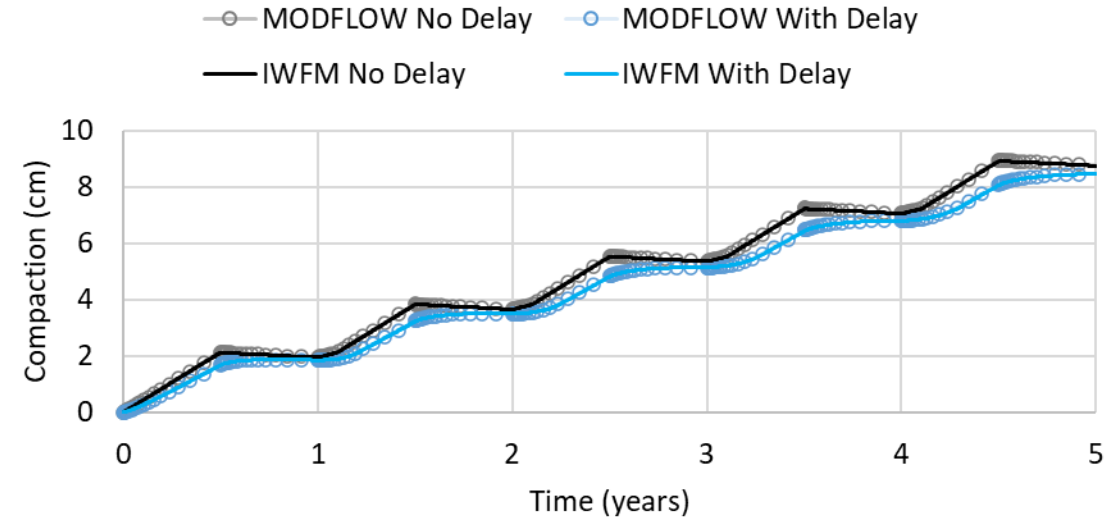
# Example 2

- MODFLOW and IWFM models simulated using no-delay (original IWFM) formulation



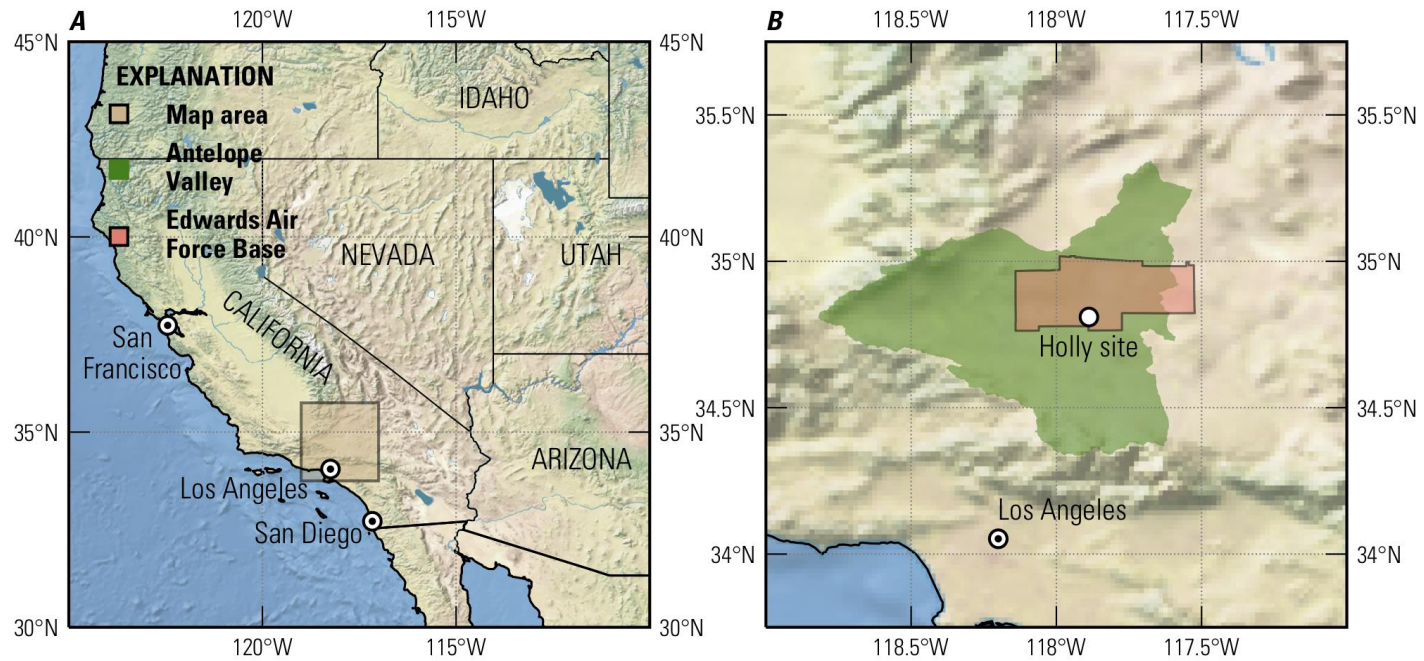
# Example 2

- MODFLOW and IWFM models simulated using with-delay formulation
- Compaction is not impacted as much as head change
- Head change is much pronounced with delayed subsidence



# Example 3

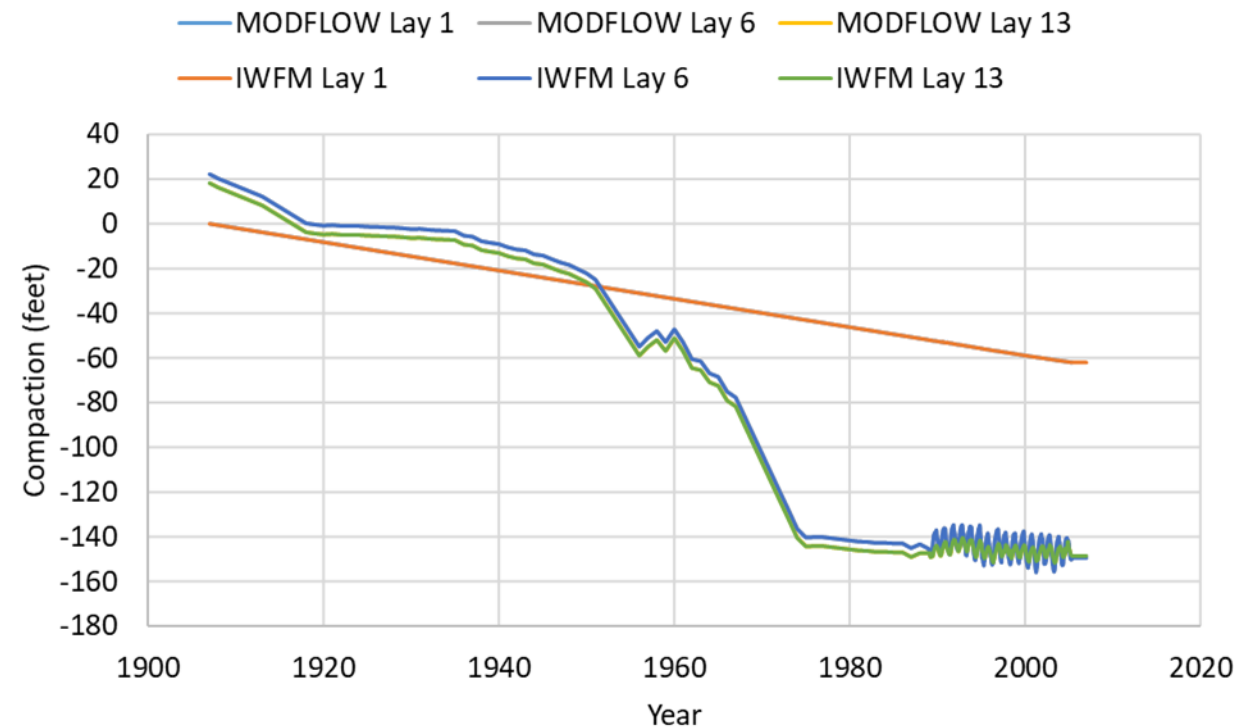
- One-dimensional model for Holly Site, Antelope Valley, CA





# Example 3

- MODFLOW and IWFM impose a specified head boundary and monitor subsidence

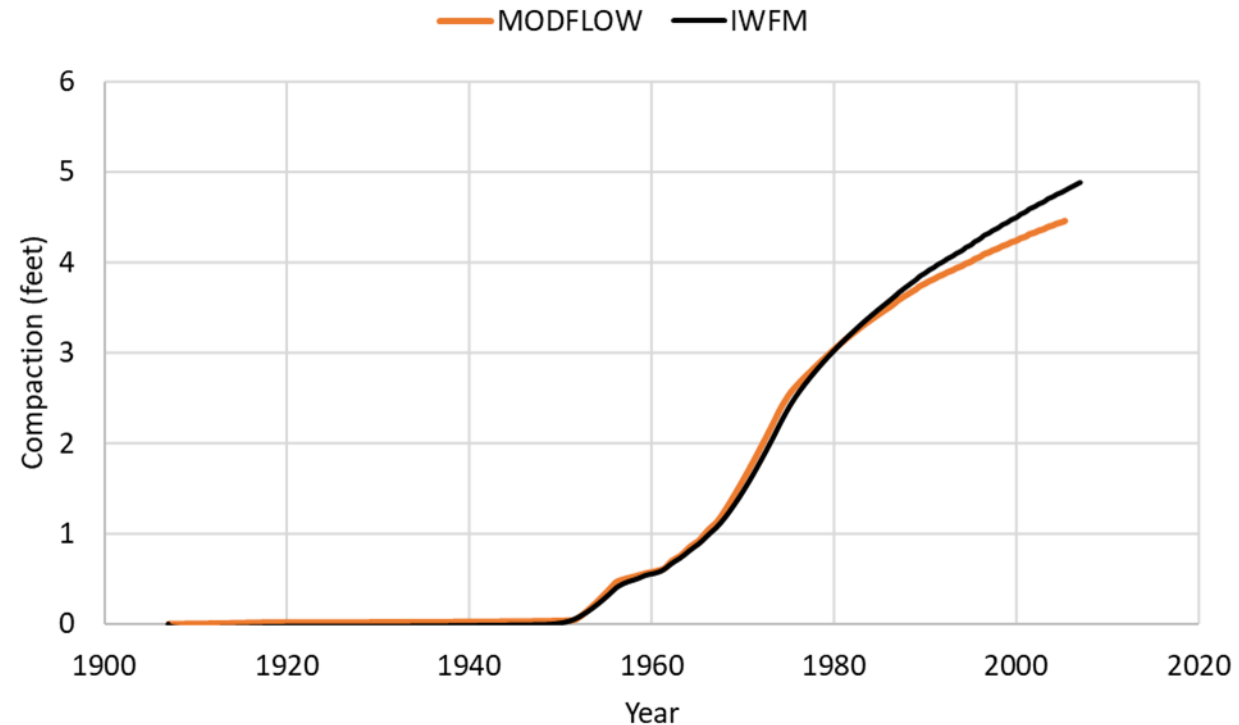




# Example 3

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- MODFLOW uses mixed delayed and no-delay interbeds
- MODFLOW calibrated values to match observed values
- IWFM used delayed interbed response



# Discussion

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# Confined Flow Equation

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- In a confined flow equation, specific storage is used to represent the change in storage and lumps together:
  - Water compressibility
  - Aquifer compressibility
- Specific storage is:
  - Inherently elastic
  - Instantaneous

$$S_s \frac{\partial h}{\partial t} = -\nabla \cdot (-K \nabla h) - G$$

# Subsidence Formulation

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- Elastic and inelastic storage within clay interbeds
- Delayed (time-dependent) response of storage within clay interbeds

$$S_s \frac{\partial h}{\partial t} = -\nabla \cdot (-K \nabla h) - G$$

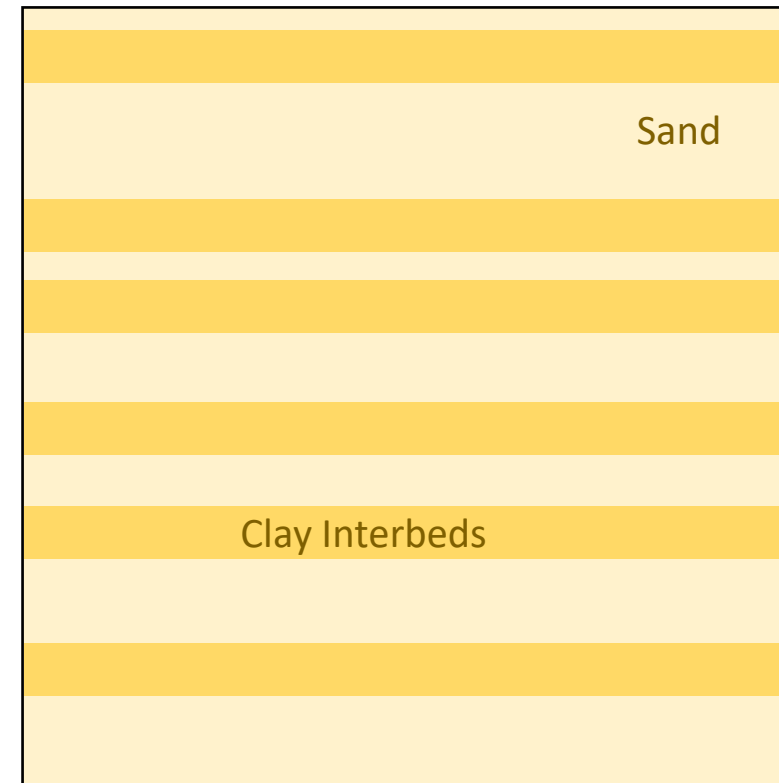
$$K \nabla^2 h = S_{sw} \frac{\partial h}{\partial t} + \frac{\partial}{\partial t} \nabla \cdot \mathbf{u},$$

# Subsidence Formulation

- Coarse-grained material may be considered as ‘aquifer’ and only elastic storage used
  - This term would be assigned as the specific storage ( $S_s$ ) term
- Clay interbeds are simulated as part of the subsidence package.
  - Elastic ( $S_{ke}$ ) and inelastic ( $S_{kv}$ ) specific storage for fine-grained material becomes input for the subsidence package

$$S_s \frac{\partial h}{\partial t} = -\nabla \cdot (-K\nabla h) - G$$

$$K\nabla^2 h = S_{sw} \frac{\partial h}{\partial t} + \frac{\partial}{\partial t} \nabla \cdot \mathbf{u},$$



# Path Forward

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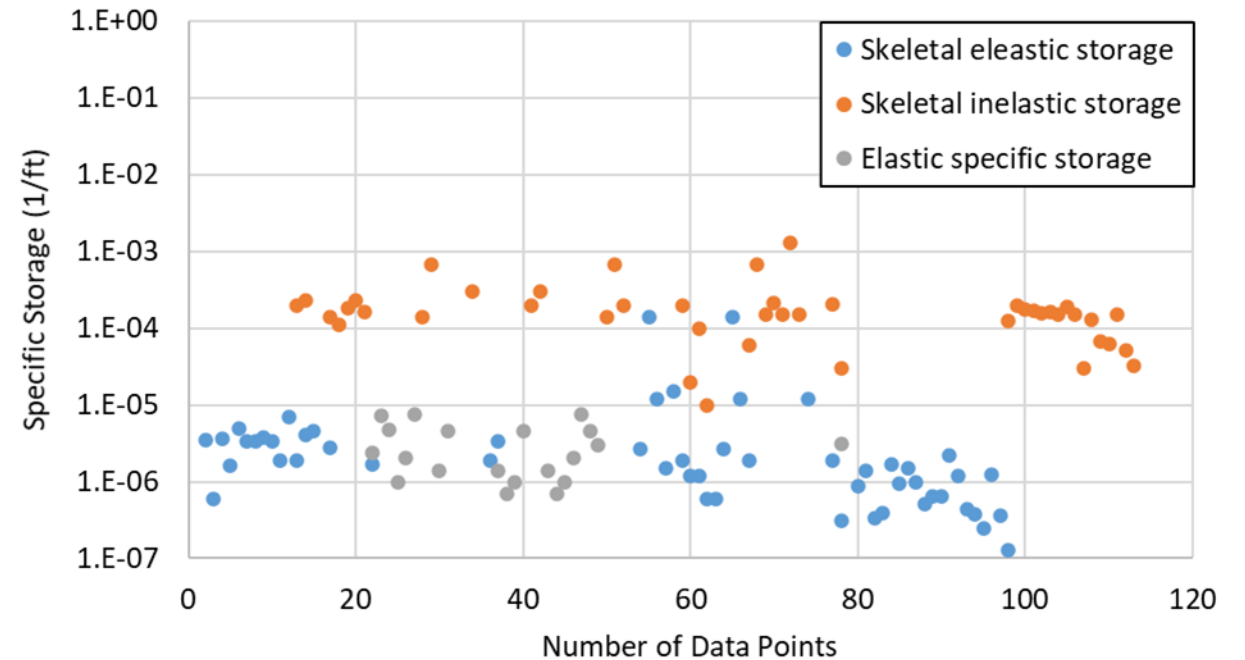
# C2VSim-FG Application

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- Incorporate delayed change in storage and subsidence in C2VSim-FG
- Calibrate the model with subsidence observations
- Update Texture2Par as necessary

# Central Valley

- Data compiled from more than twenty Central Valley publications
- Specific storage values approximately fall within two ranges
  - Inelastic storage  $10^{-4}$  to  $10^{-3}$  /foot
  - Elastic storage  $10^{-7}$  to  $10^{-5}$  /foot





Thank you for your time!

Questions?

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