A New Vadose Zone Model for IWFM

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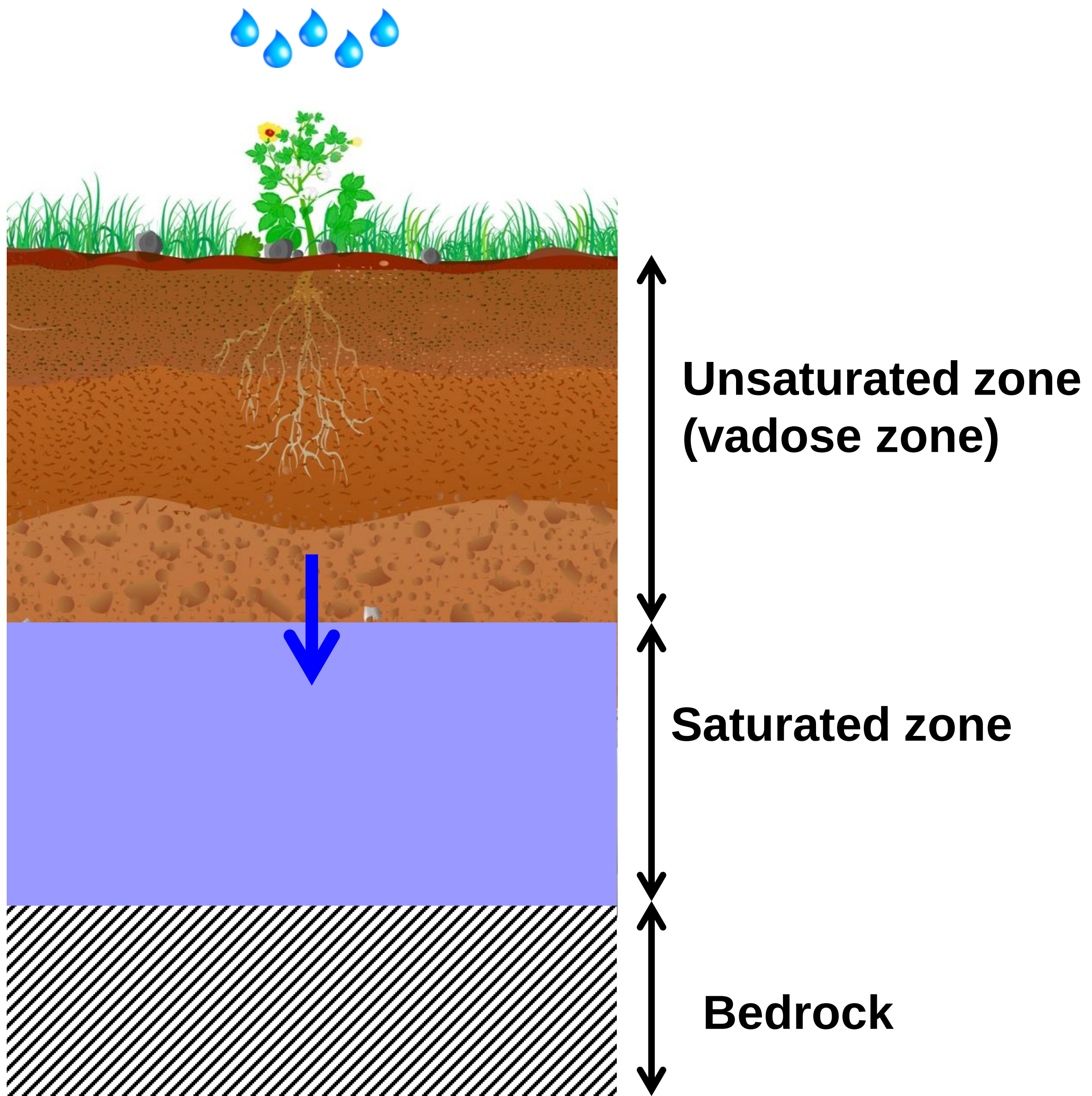
Motivation

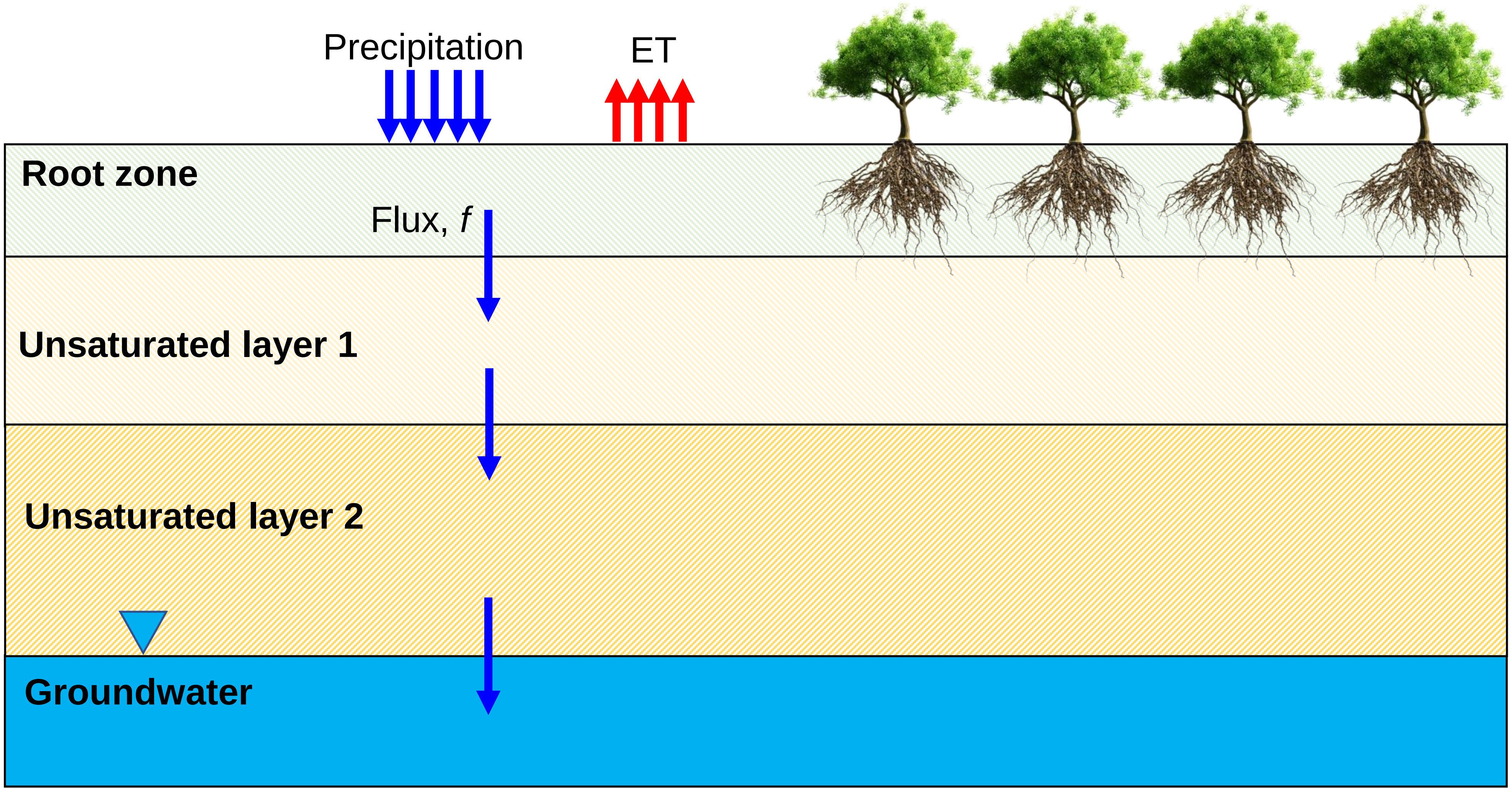
1) What is groundwater recharge?

2) Can recharge be negative (backward flow to vadose zone)?



Motivation

- Vadose zone flux in IWFM is always downward due to gravity.
- Kinematic wave solution (MODFLOW family) is similar.



Objective

Reference model:

$$f = -D(\theta)\frac{\partial\theta}{\partial z} + K(\theta)$$

Accurate ✓
Computationally demanding X

Current groundwater models:

$$f = K(\theta)$$

Computationally efficient ✓
Not very accurate X

f: water flux
θ: soil moisture
z: soil depth
K: unsaturated conductivity
D: soil water diffusivity

Target model:

$$f(\theta) = ?$$

Accurate ✓
Computationally efficient ✓

Challenge

Darcy law

$$f = -D\frac{\partial \theta}{\partial z} + K$$

Conservation of mass

$$\frac{\partial \theta}{\partial t} = -\frac{\partial f}{\partial z}$$

Richards equation

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[D\frac{\partial \theta}{\partial z}\right] - \frac{\partial K}{\partial z}$$

- Highly nonlinear equation
- Arbitrary soil hydraulic functions, $K(\theta)$ and $D(\theta)$
- Variable surface boundary conditions
- Variable groundwater table
- …

Key Idea

$$\frac{\partial \theta}{\partial t} = D \frac{\partial^2 \theta}{\partial z^2} \quad \Rightarrow \quad \frac{\partial \theta}{\partial z} = \sqrt{\frac{1}{D}} \frac{\partial^{0.5} \theta}{\partial t^{0.5}}$$

Gradient from single-level soil moisture data ✓

Proposed Model

Flux from single-level soil moisture data ✓

$$f = -\sqrt{cD(\theta)}H(\theta) + K_s\left[\frac{K(\theta) - K(h = DWT)}{K_s - K(h = DWT)}\right]$$

$$H = \frac{\partial^{0.5}\theta}{\partial t^{0.5}} \approx -\frac{1}{\sqrt{\pi}}\sum_{i=1}^{N-1}\frac{\theta(\tau_{i+1}) - \theta(\tau_i)}{\sqrt{t - 0.5(\tau_i + \tau_{i+1})}}$$

Model Evaluation

- HYDRUS-1D simulations in Tonzi ranch

- Van Genuchten soil hydraulic functions

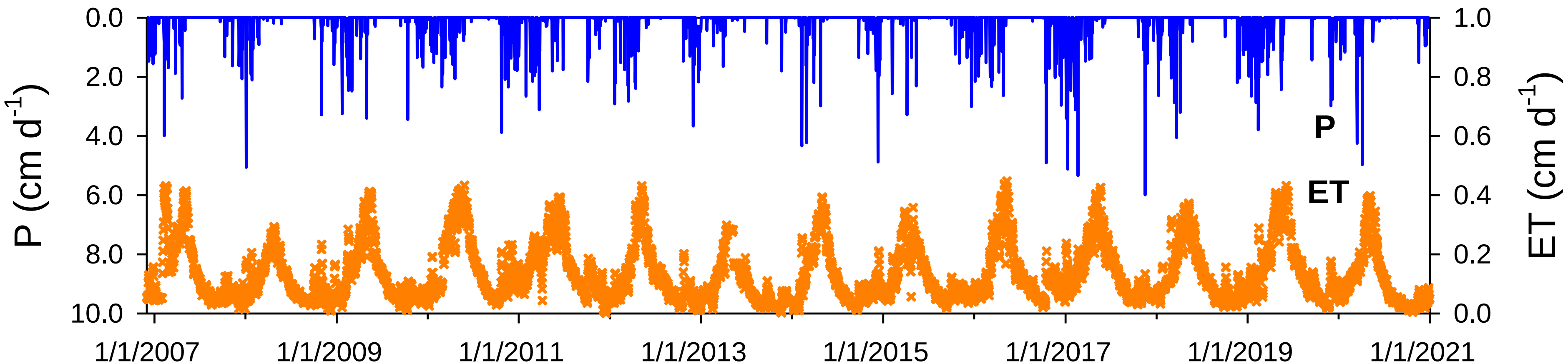
$$\Theta = [1 + (\alpha h)^n]^{-m}$$

$$K = K_s \Theta^{0.5} \left[1 - \left(1 - \Theta^{1/m} \right)^m \right]^2$$

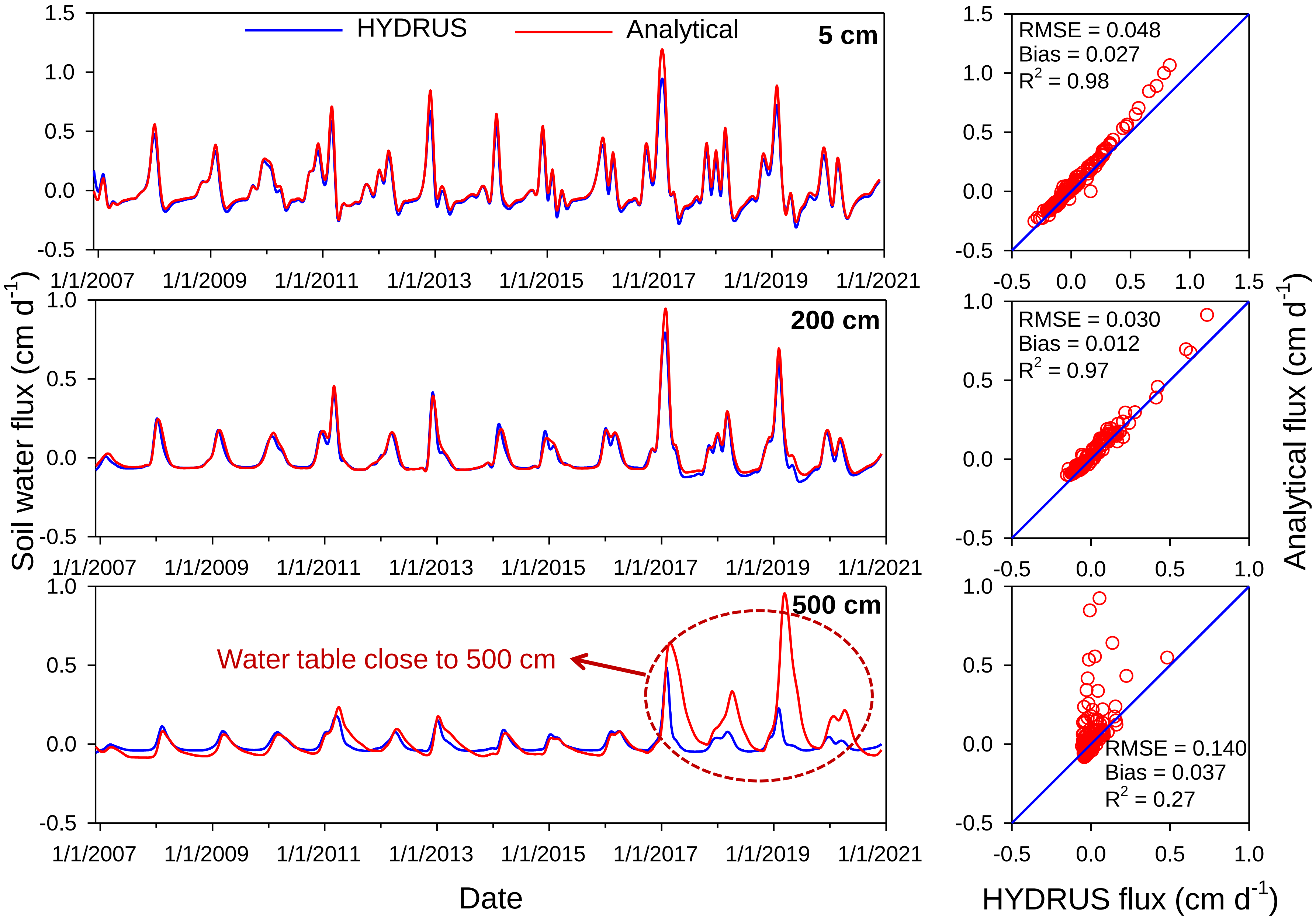
$$\Theta = (\theta - \theta_r) / (\theta_s - \theta_r)$$

Tonzi Ranch AmeriFlux site, California

- Top boundary from P & ET observations



Sample Flux Results



More Details

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Research papers

Estimating soil water flux from single-depth soil moisture data

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ARTICLE INFO

ABSTRACT

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Keywords:

Combining Eqs. (15) and (16), we get our final solution to the vadose zone water flux as follows:

$$f \approx -\sqrt{cD}H + K_s\left(\frac{K-K_d}{K_s-K_d}\right) \quad (17)$$

The first term of Eq. (17) is due to the soil moisture diffusion, which

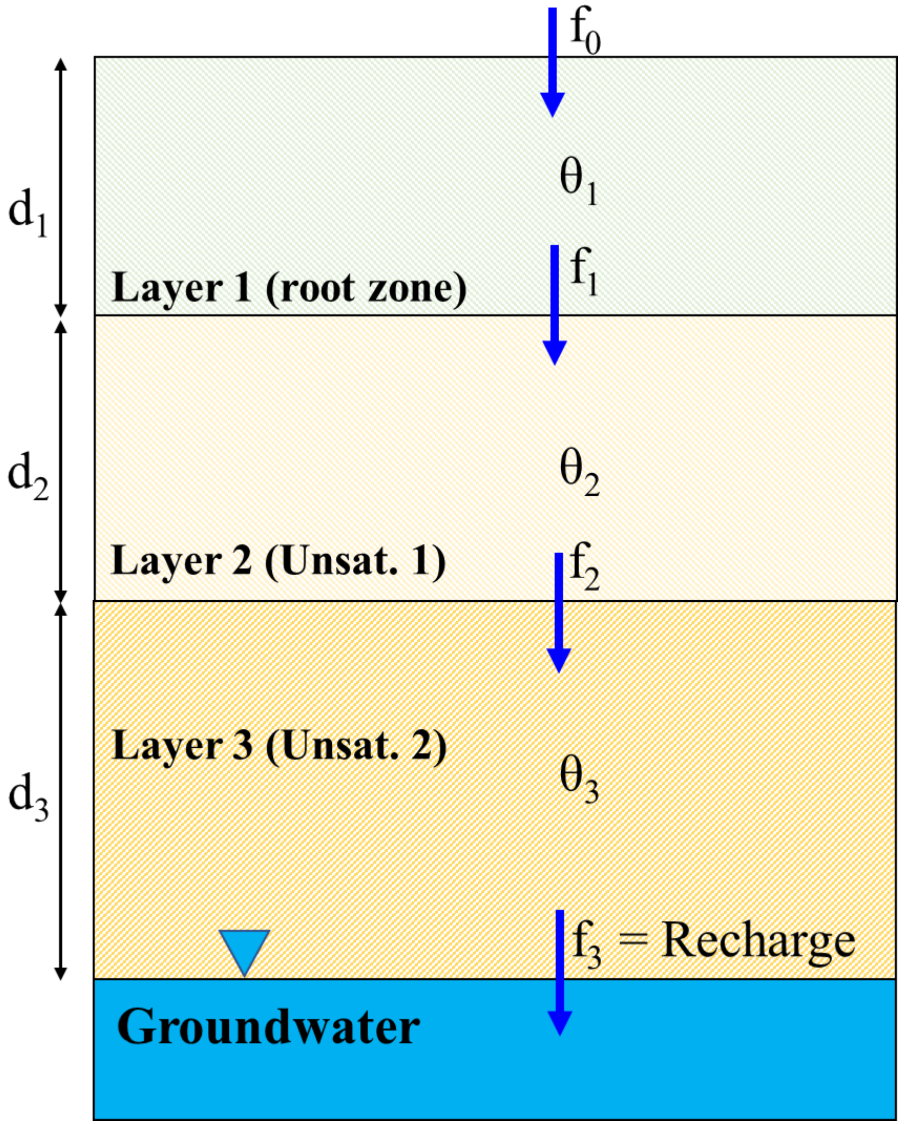
Soil Moisture Routing

Current equation (one-way flux):

$$f = K(\theta)$$

Proposed equation (two-way flux):

$$f = -\sqrt{cD(\theta)}H(\theta) + K_s\left[\frac{K(\theta) - K(DWT)}{K_s - K(DWT)}\right]$$



Soil Moisture Routing

Solving for layer 1:

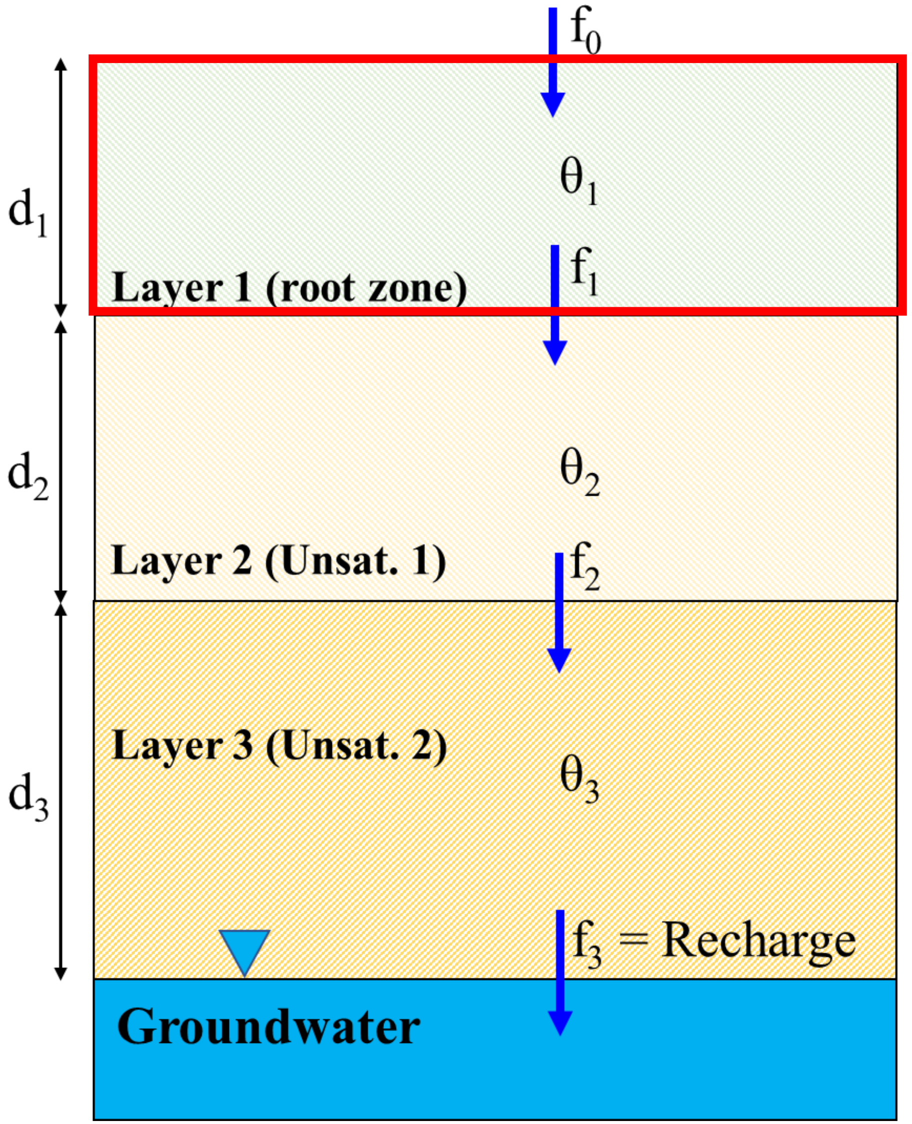
$$f_1 = f_0 - (\theta_1 - \theta_{i1})\frac{d_1}{\Delta t} \quad \textbf{(A)}$$

$$\frac{f_0 + f_1}{2} = -\sqrt{cD_1(\theta_1)H(\theta_1)} + K_{s1}\left[\frac{K_1(\theta_1) - K_1(DWT)}{K_{s1} - K_1(DWT)}\right] \quad \textbf{(B)}$$

$$f_0 = 0.5(\theta_1 - \theta_{i1})\frac{d_1}{\Delta t} - \sqrt{cD_1(\theta_1)H(\theta_1)} + K_{s1}\left[\frac{K_1(\theta_1) - K_1(DWT)}{K_{s1} - K_1(DWT)}\right] \quad \textbf{(C)}$$

Solving Eq. (C) yields θ_1.

Then, f_1 will be solved using Eq. (A).



Soil Moisture Routing

Solving for layer 2:

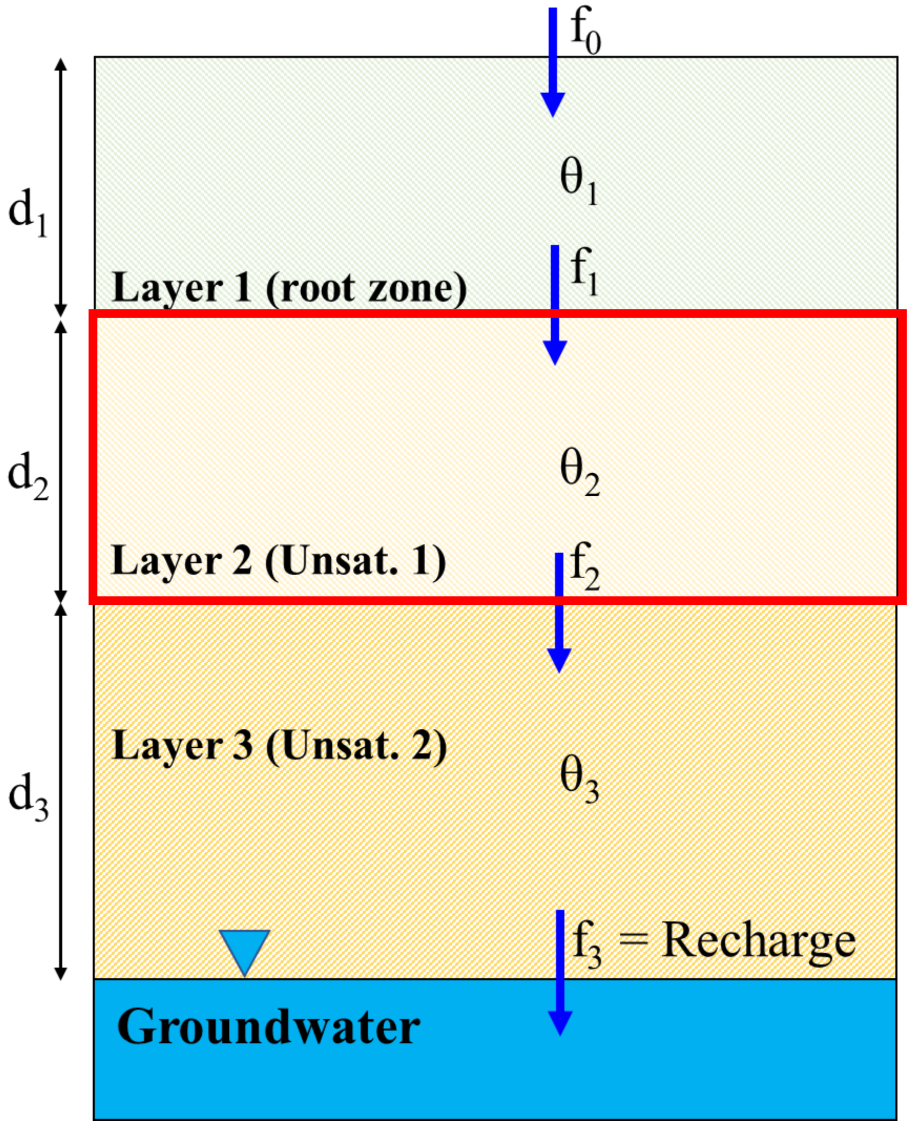
$$f_2 = f_1 - (\theta_2 - \theta_{i2})\frac{d_2}{\Delta t} \quad \textbf{(A)}$$

$$\frac{f_1 + f_2}{2} = -\sqrt{cD_2(\theta_2)}H(\theta_2) + K_{s2}\left[\frac{K_2(\theta_2) - K_2(DWT)}{K_{s2} - K_2(DWT)}\right] \quad \textbf{(B)}$$

$$f_1 = 0.5(\theta_2 - \theta_{i2})\frac{d_2}{\Delta t} - \sqrt{cD_2(\theta_2)}H(\theta_2) + K_{s2}\left[\frac{K_2(\theta_2) - K_2(DWT)}{K_{s2} - K_2(DWT)}\right] \quad \textbf{(C)}$$

Solving Eq. (C) yields θ_2.

Then, f_2 will be solved using Eq. (A).



Soil Moisture Routing

Solving for layer 3:

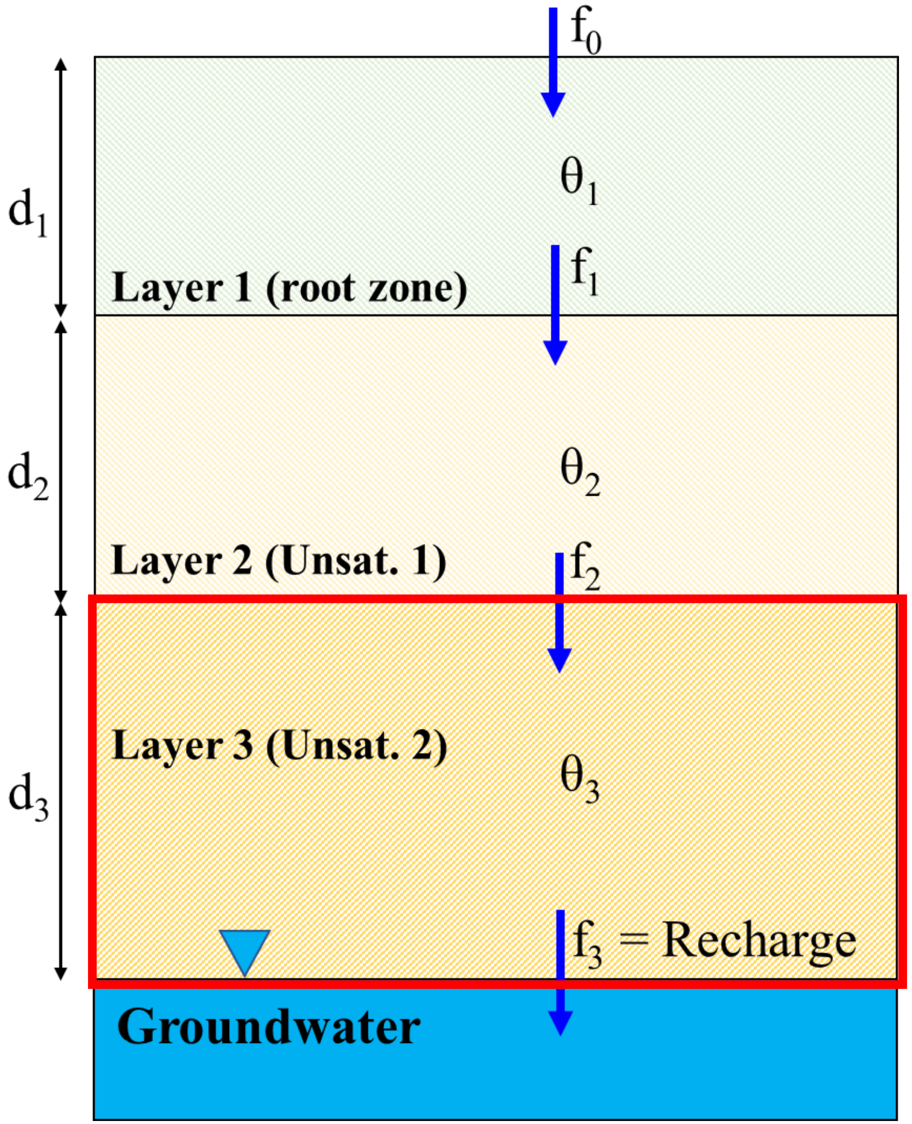
$$f_3 = f_2 - (\theta_3 - \theta_{i3})\frac{d_3}{\Delta t} \quad \textbf{(A)}$$

$$\frac{f_2 + f_3}{2} = -\sqrt{cD_3(\theta_3)}H(\theta_3) + K_{s3}\left[\frac{K_3(\theta_3) - K_3(DWT)}{K_{s3} - K_3(DWT)}\right] \quad \textbf{(B)}$$

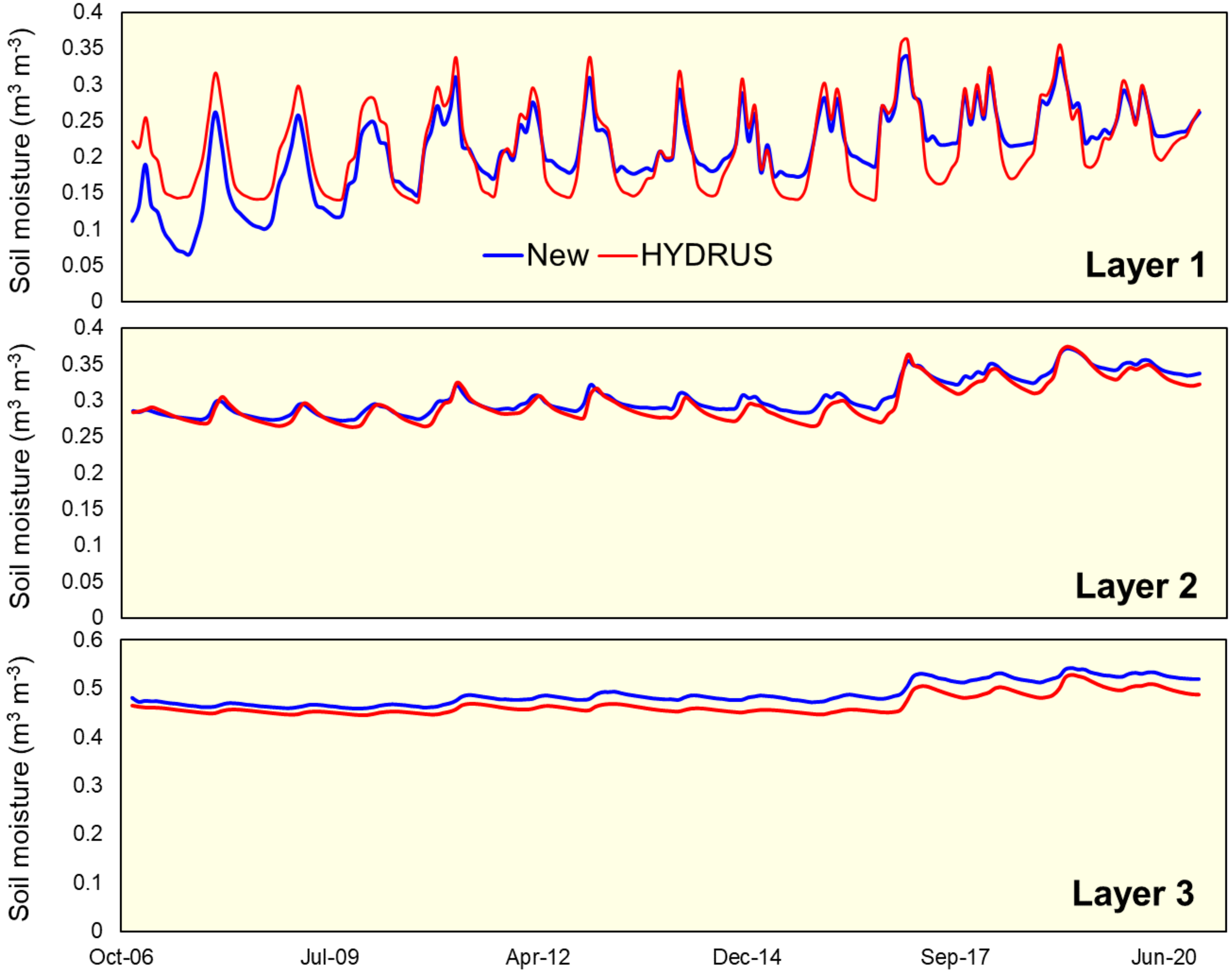
$$f_2 = 0.5(\theta_3 - \theta_{i3})\frac{d_3}{\Delta t} - \sqrt{cD_3(\theta_3)}H(\theta_3) + K_{s3}\left[\frac{K_3(\theta_3) - K_3(DWT)}{K_{s3} - K_3(DWT)}\right] \quad \textbf{(C)}$$

Solving Eq. (C) yields θ_3.

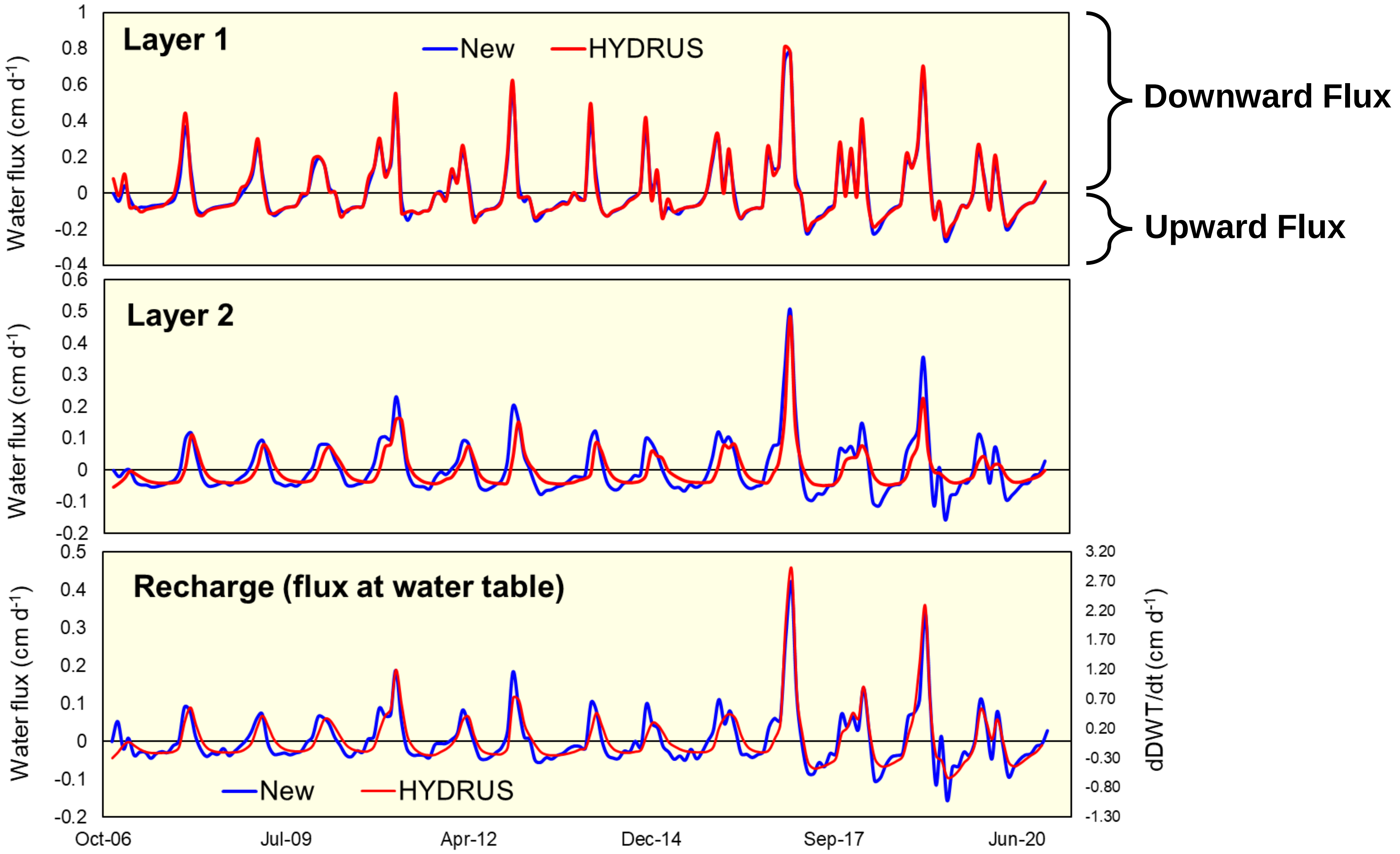
Then, f_3 will be solved using Eq. (A).



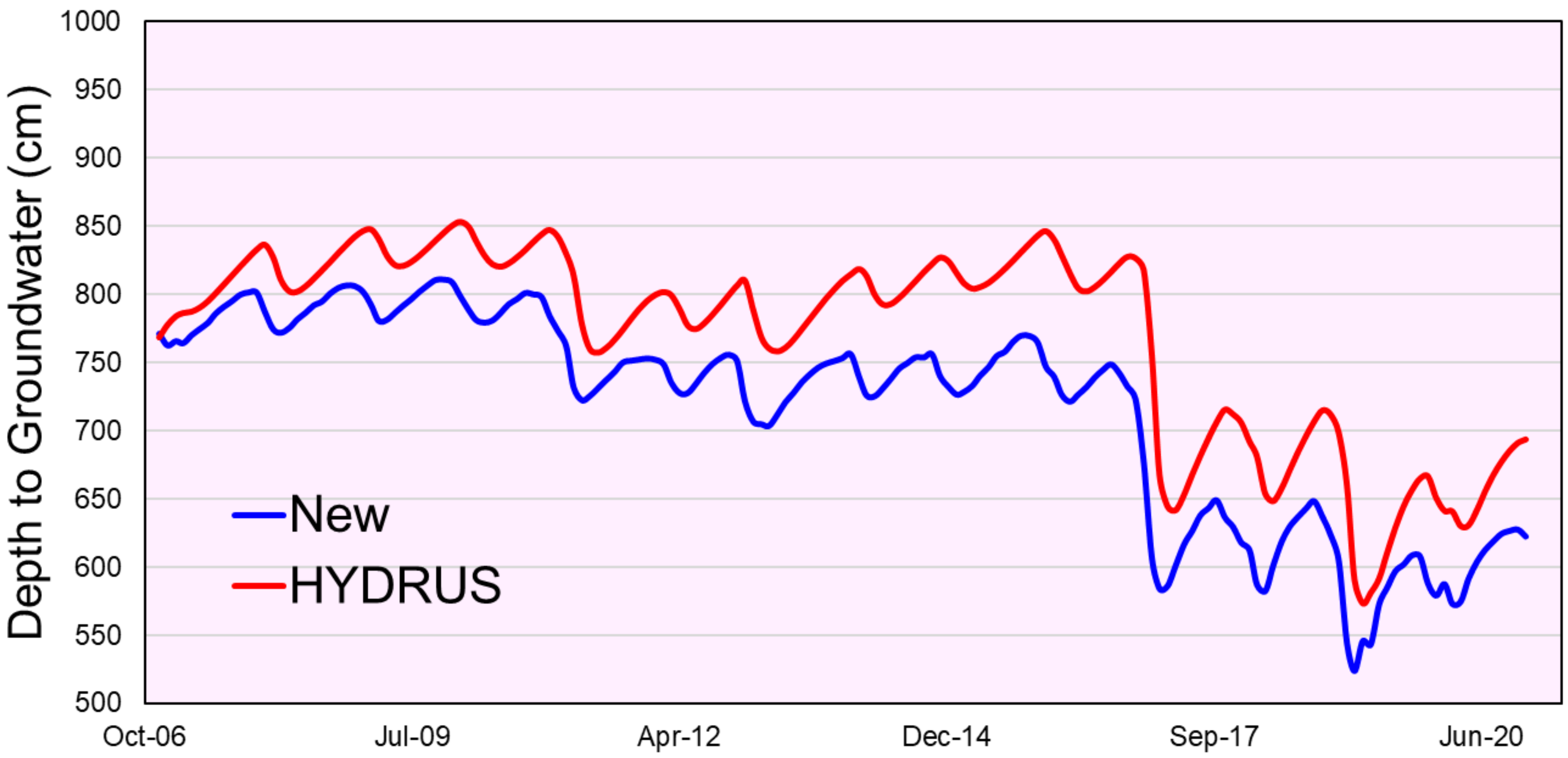
Modeled Soil Moisture



Modeled Flux



Modeled Groundwater Level



Future Perspective

- Testing the algorithm for more scenarios
- Employing the new model in IWFM / C2VSim-FG

CALIFORNIA DEPARTMENT OF WATER RESOURCES
SUSTAINABLE GROUNDWATER
MANAGEMENT OFFICE

Thank you!

Questions & Comments:

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